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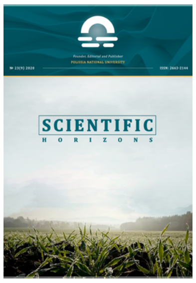

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Improving Control Accuracy in Multi-Connected Digital Systems

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Abstract. The presented scientific research is relevant, because currently it is necessary to develop and implement modern control systems for technological processes. This allows increasing the accuracy of control in multi-connected digital systems, the mathematical models of which are built on the platform of the state space method. The purpose of this study is to develop a new method for improving control accuracy in multi-connected digital systems. The methodological framework of this study, determined directly for the qualitative solution of the problem, included analytical expressions that functionally not only eliminate the influence of each state and control on the rest, but also ensure high accuracy of control processes. In this scientific study, the results were obtained, standing for a methodical approach to the synthesis of vector-matrix models of regulators using feedback on the state. The vector-matrix model of the controller, combining the function of monitoring and control of feedback on the state was formed. By using computational capabilities of the mathematical apparatus adopted in the study, the matrices of the system regulators and correction coefficients were calculated. The formulated conclusions affect various aspects of the practical application of the method of determining the feedback matrix by state, deriving an analytical formula for determining the correction coefficients to ensure zero steady-state control error, as well as performing the decomposition of a digital system with the definition of a vector-matrix model of a regulator combining the functions of regulation and supervision. The materials and methods of paper fully correspond to the stated subject and can serve as a qualitative methodological basis for following research in this area

Keywords: state space method, quality of control processes, vector-matrix models, feedback

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INTRODUCTION

Nowadays, there are many publications (Lenzen & Vollmering, 2020; Wen *et al.*, 2019) where the authors solve the problems of creating control systems based on the state space method. Each author finds their niche for solving urgent problems of developing and researching various classes of control systems on the mathematical platform of the state space method, synthesis of multidimensional systems. The scientific monographs on the presentation of a wide range of tasks for the design of vector-matrix models of digital systems are published (Repnikova, 2017). This article uses a well-founded method for the synthesis of feedback systems (Fadali & Visioli, 2019). But if for continuous systems there are clear recommendations for finding the desired roots, then for digital systems, if such recommendations exist – they are not generalized, and a control error is observed as a result of synthesis.

The problems of pragmatic, structural synthesis of automatic control systems for technological processes occurring in various environments and digital systems are found by criteria of multidimensionality, multi-connectivity, as well as the nonlinearity of mathematical models of these systems (Yang & Sun, 2021). Considering the specified characteristics of the designated objects, it should be noted that until recently, the management of complex technological processes supported the practical application of traditional linear systems formed based on standard mathematical algorithms. The synthesis of systems of linear type, whose operation is based on the practical application of the input and output models, does not assume the accounting multidimensionality, and the relative position of the coordinate system status and influence on the process itself. An increasing influence of the signal and parametric perturbations is on the quality of initial management processes in systems of this kind (Wen *et al.*, 2019). In a situation where there is a constant increase in the volume of requirements for the efficiency of management of modern technological processes, it should be considered fully justified and technologically expedient to use control principles using vectors of the object's state. While considering the use of inertia-free state regulators in calculations, as well as joint regulators of this state.

Today, a prerequisite for obtaining a qualitative and objective solution to the problem of modeling a control system for a multidimensional object based on the principles of managing its states should be considered obtaining a qualitative solution to the problem of parametric and structural synthesis of automated control systems for multi-connected, nonlinear, and multilevel objects. A qualitative solution to this problem has not yet been obtained that can me*et al*l the technological requirements of the construction of a multidimensional object management system (Giraud & Giraud-Audine, 2019). This dictates the importance and significant prospects for finding effective algorithms for controlling multi-connected digital systems and improving the accuracy of this control, which is essential from the point of view of practical operation of modern technological devices. The control systems that exist today show high efficiency with stable parameters of the object, the absence of a clear structural relationship between its basic states and without pronounced links to the quality of its management. In the presence of any changes in the state of the object and the loss of relationships between them, the control system malfunctions, due to the inability to control processes in the changed multidimensional conditions (Szederkenyi *et al.*, 2018).

The main purpose of this study is to develop a new method for the synthesis of multi-connected digital systems. This technique is based on the analytical derivation of an expression to determine the correction coefficients that ensure high quality control with subsequent modelling. Such a technique does not need the task of figuring out the specific values of the desired roots, which simplifies its use for practical engineers when creating digital control systems of any field of use.

MATERIALS AND METHODS

The basis of the methodological approach in this research work is the practical application of a set of analytical expressions that functionally not only eliminate the influence of each state and control on the rest, but also ensure high accuracy of control processes. The basic procedure for conducting scientific research was preceded by the creation of a qualitative theoretical base, which is the basis for the later implementation of the chosen model of scientific research, involving the appropriate mathematical platform of research methods.

Theoretical framework of this study included the investigation of the available publications of several Ukrainian and international researchers on problems directly or indirectly related to issues of improving control accuracy in multi-connected digital systems, with the involvement of an optimal mathematical platform for implementation of the entire set of research tasks.

To achieve this task, the study was performed in five areas using the following methods:

- a mathematical platform of state-space method using the classical theory of synthesis of systems with feedback;
- attraction analytical formulas for junction channels of R. Iserman (2018);
- conclusion analytical expressions for the correction factors to ensure zero steady-state error control using a theorem of Z-transform on finite value of the function;
- the use of monitored devices;
- attraction formula decomposition of the system with the calculation of controller that combines the functions of control and observation.

The developed technique was tested on models using the Matlab/Simulink application package for control objects whose dynamics are described by vector-matrix equations of state and output.

During this scientific research, scientific experiments were carried out designed to simulate: transients in the initial multi-connected digital system, transients with a feedback controller, transients in the initial multi-connected digital system, transients with a feedback controller, transients using the derived formula for calculating correction coefficients, as well as transients in a system with a combined controller providing high control accuracy. The three experiments were accompanied by the display of graphical dependencies of the quantities obtained during the simulation of the processes under consideration, as well as digital display of the parameters obtained during their simulation.

RESULTS AND DISCUSSION

The study of the main problematic aspects of improving control accuracy in multi-connected digital systems has given the following results. Multidimensional digital systems are considered, which are described by equations (1), (2):

$$\mathrm{x[n+1]=Ax[n]+Bu[n]}, \tag{1}$$

$$\mathrm{y[n]=Cx[n]}, \tag{2}$$

where A, B, C are the diagonal matrices; x, y, u are the initial variables; n is the reference number.

The general view of the matrices of a multi-connected object (3-5):

$$A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}, \tag{3}$$

$$B = \begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix}, \tag{4}$$

$$C = \begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix}. \tag{5}$$

The equation of a closed digital system x[n+1]==[A-BK]x[n]=Fx[n], where K is the state feedback matrix. A multidimensional system with a state controller, the initial variables of which do not affect each other, is described by the following Iserman equation (6):

$$\mathrm{y[n+1]=Ty[n]+TCx[n]} \tag{6}$$

where $\mathrm{T}=diag(z_i)_{\mathrm{i}}=1,2,\ldots 1$.

For the moment of reference [n+1], one can write (7):

$$\mathrm{y[n+1]=Cx[n+1]=C[A\text{-}BK]x[n]}. \tag{7}$$

From where the equation of the state regulator is obtained (8):

$$\mathrm{K=[CB]^{1}\,[CA\text{-}TC]}. \tag{8}$$

For an n^th order system, the regulator matrix (8) for diagonal matrices B and C is calculated as (9):

$$\mathrm{K} = \begin{bmatrix} \frac{a_{11}-z_1}{b_{11}} & \frac{a_{12}}{b_{11}} \cdots \frac{a_{1n}}{b_{11}} \\ \frac{a_{21}}{b_{22}} & \frac{a_{22}-z_2}{b_{22}} \cdots \frac{a_{2n}}{b_{22}} \\ \frac{a_{n1}}{b_{nn}} & \frac{a_{n2}}{b_{nn}} \cdots \frac{a_{nn}-z_n}{b_{nn}} \end{bmatrix} \tag{9}$$

where z_i are the desired roots.

Studies have shown that although such a technique provides dilution of control channels by condition, there is a regulatory error. To eliminate the control error, a formula for determining the correction coefficients is proposed. To derive the formula for finding the correction coefficients, the z-transformation theorem of the final value of the function was used. Authors derive an analytical dependence for the matrix of correction coefficients K* with dimension *lxl* of a multidimensional digital system (10):

$$\mathrm{K}^* = \begin{bmatrix} k_{11}^* \ldots k_{1l}^* \\ \ldots\ldots\ldots. \\ k_{l1}^* \ldots\ k_{ll}^* \end{bmatrix} \tag{10}$$

For this, authors define the matrix transfer function of a closed control system (11).

$$W_3^*(z) = \frac{Y(z)}{U(z)} = \mathrm{C(zE - A + B)^{-}B}, \tag{11}$$

where Y, U are the initial variables; z is the desired root.

The expression $\mathrm{W_w}\,(z)$ was written in parentheses for the original two-dimensional system in matrix form (12), (13).

$$\begin{aligned} &\left(z\cdot\begin{pmatrix}1\,0\\0\,1\end{pmatrix} - \begin{pmatrix}a_{11}\,a_{12}\\a_{21}\,\ a_{22}\end{pmatrix} + \begin{pmatrix}b_{11}\,b_{12}\\b_{21}\,\ b_{22}\end{pmatrix}\cdot\begin{pmatrix}k_{11}\,k_{12}\\k_{21}\,\ k_{22}\end{pmatrix}\right)^{-1} = \\ &= \left(\begin{pmatrix} z-\ a_{11} & -\ a_{12} \\ -\,a_{21} & z-\ a_{22} \end{pmatrix} + \begin{pmatrix} b_{11}\cdot\ k_{11}+b_{12}\cdot\ k_{21} & b_{11}\cdot\ k_{12}+b_{12}\cdot\ k_{22} \\ b_{21}\ \cdot\ k_{11}+b_{22}\cdot\ k_{21} & b_{21}\ \cdot\ k_{12}+b_{22}\cdot\ k_{22} \end{pmatrix}\right)^{-1} = \\ &= \begin{pmatrix} z-a_{11}+b_{11}\cdot k_{11}+b_{12}\cdot k_{21} & -\,a_{12}+b_{11}\cdot k_{12}+b_{12}\cdot k_{22} \\ -a_{21}+b_{21}\cdot k_{11}+b_{22}\cdot k_{21} & z-a_{22}+b_{21}\cdot k_{12}+b_{22}\cdot k_{22} \end{pmatrix}^{-1} = \\ &= \begin{pmatrix} \dfrac{z-a_{22}+b_{21}\cdot k_{12}+b_{22}\cdot k_{22}}{D} & \dfrac{a_{12}-b_{11}\cdot k_{12}+b_{12}\cdot k_{22}}{D} \\ \dfrac{a_{21}-b_{21}\cdot k_{11}+b_{22}\cdot k_{21}}{D} & \dfrac{z-a_{11}+b_{11}\cdot k_{11}+b_{12}\cdot k_{21}}{D} \end{pmatrix} \end{aligned} \tag{12}$$

$$\begin{aligned} D(z) = z^2 &+ (-a_{22}+b_{21}\cdot k_{12}+b_{22}\cdot k_{22})\cdot(-a_{11}+b_{11}\cdot k_{11}+b_{12}\cdot k_{21} \\ &\cdot z + (-a_{22}+b_{21}\cdot k_{12}+b_{22}\cdot k_{22})\cdot(-a_{11}+b_{11}\cdot k_{11}+b_{12}\cdot k_{21}) + \\ &+(-a_{12}+b_{11}\cdot k_{12}+b_{12}\cdot k_{22})\cdot(-a_{21}+b_{21}\cdot k_{11}+b_{22}\cdot k_{21}). \end{aligned} \tag{13}$$

Since the following equality is fulfilled in the synthesized system by a matrix of K coefficients of linear stationary feedbacks (14):

$$\det[zE\text{-}A\text{+}BK]=\det[zE\text{-}F], \tag{14}$$

where F is the transition matrix of the state of the closed system.

That is, $D(z)=D^*(z)=(z\text{-}z_1)(z\text{-}z_2)$ one can replace the expression for the characteristic polynomial of the original closed-loop system is $D(z)$ as the desired characteristic polynomial is of the form: $D^*(z)=(z\text{-}z_1)(z\text{-}z_2)$. Because of the performed substitution, authors obtain the following expression for the matrix transfer function of a closed system (the case of diagonal matrices C and B) (15), (16).

$$W(z)=\begin{vmatrix}\dfrac{c_{11}b_{11}}{z-z_1} & 0\\ 0 & \dfrac{c_{22}b_{22}}{z-z_2}\end{vmatrix}=K_h\begin{vmatrix}\dfrac{1}{z-z_1} & 0\\ 0 & \dfrac{1}{z-z_2}\end{vmatrix} \tag{15}$$

For the general case of an n-dimensional system:

$$K_h=\begin{vmatrix}k_{h11} & 0\,\dots & 0\\ 0 & k_{h22}\,\dots & 0\\ 0 & 0\,\dots & k_{hlm}\end{vmatrix}k_{hij}=c_{ii}b_{jj}, \tag{16}$$

where h, i, j are the system state variables.

Authors determine the final values of the vector function of the output of the system under study $y[n]$, with n tending to infinity. For this, the following formula to calculate the final value of the function was used (17):

$$\lim_{n\to\infty}y[n]=\lim_{z\to1}\frac{z-1}{z}Y(z) \tag{17}$$

With an incoming action represented as $g(z)=z/z\text{-}1$, the function $Y(z)$ can be represented as (18):

$$Y(z)=W_3(z)\frac{z}{z-1}, \tag{18}$$

then (19),

$$y[n]=\lim_{z\to1}\frac{z-1}{z}W_3(z)\frac{z}{z-1}\mathrm{TK}^*\mathrm{K}_h\begin{vmatrix}\dfrac{1}{z-z_1} & 0\\ 0 & \dfrac{1}{z-z_2}\end{vmatrix}=\mathrm{E}, \tag{19}$$

To ensure zero adjustment error, the matrix of correction coefficients K^*, considering formulas, is calculated by the formula (20):

$$\mathrm{K}^*=T^{-1}K^{-1}\begin{vmatrix}\dfrac{1}{z-z_1} & 0\\ 0 & \dfrac{1}{z-z_2}\end{vmatrix}^{-1}=E \tag{20}$$

To generalize the method and the logical finale, authors use general analytical formulas for the synthesis of the observed device (21):

$$\begin{gathered}\mathrm{A}_{CP}=[\mathrm{A}-\mathrm{HC}]\\ \mathrm{B}_{CP}=[\mathrm{B}\ \ \mathrm{H}]\\ \mathrm{C}_{CP}=[\mathrm{E}]\\ \mathrm{D}_{CP}=[0]\end{gathered} \tag{21}$$

Or the commands of the MatLab application package: $\mathrm{H}=place(A^T,C^T,P)$, where $\mathrm{P}=\{\text{-}z_1,\dots\text{-}z_n\}$ is the matrix of the desired roots of the characteristic equation. The vector-matrix model of the controller, combining the function of monitoring and control of feedback on the state, has the form (22):

$$\begin{gathered}\mathrm{A}_P=[\mathrm{A}-\mathrm{HC}-\mathrm{B}K_P]\\ \mathrm{B}_P=[\mathrm{B}\ \ \mathrm{H}]\\ \mathrm{C}_P=[-K_P]\\ \mathrm{D}_P=[1\ \ 0]\end{gathered} \tag{22}$$

The developed technique was studied for different orders of digital control systems, which were set by vector-matrix models. The trajectory of experimental studies confirming the reliability of the results obtained was created using the Matlab/Simulink package according to the following logic:

– modelling of the initial digital system (illustration of the poor quality of regulation and the influence of the states of one channel on the other);

– determination of feedback coefficients by formula (8) or (9) (illustration of the possibility of channel dilution and confirmation of the occurrence of a significant control error);

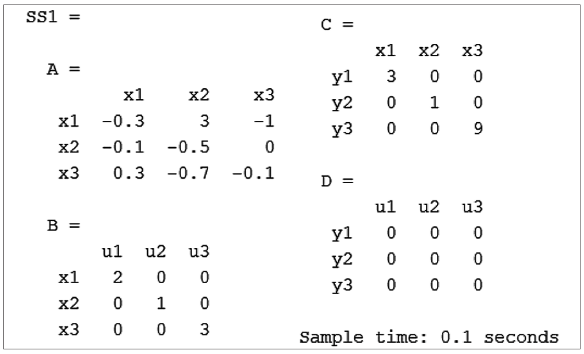
– calculation of correction coefficients according to the new formula (20) (confirmation of achievement of the scientific goal - improvement of control accuracy);

– the formation of a vector of states that cannot be obtained on real objects (21) and the definition of a vector-matrix model of a regulator combining the functions of observation and control (22), (the final steps of the proposed methodology);

– the input signal is G = (1 3 4) for all experiments.

Experiment 1.

System model (Fig. 1)

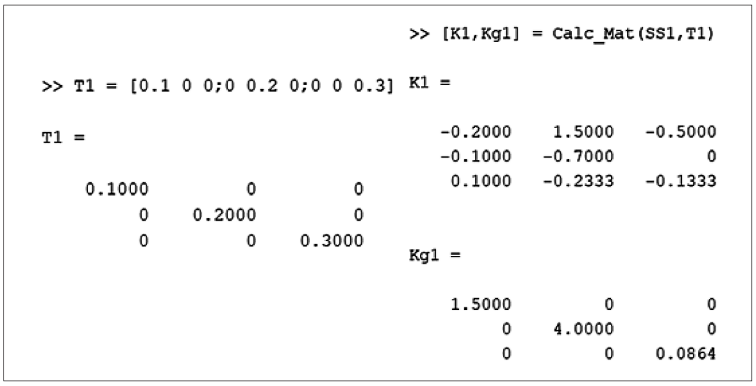


```
SS1 =                                  C =
                                              x1  x2  x3
  A =                                     y1   3   0   0
            x1      x2      x3           y2   0   1   0
    x1    -0.3       3      -1           y3   0   0   9
    x2    -0.1    -0.5       0
    x3     0.3    -0.7    -0.1         D =
                                              u1  u2  u3
  B =                                     y1   0   0   0
          u1  u2  u3                      y2   0   0   0
    x1     2   0   0                      y3   0   0   0
    x2     0   1   0
    x3     0   0   3                Sample time: 0.1 seconds
```

Figure 1. *System model*

The selected desired roots and matrices of the regulator (Fig. 2):

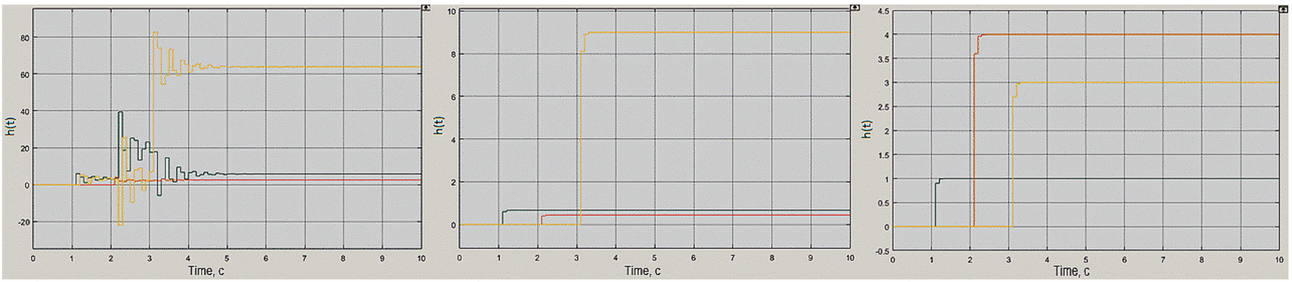
The simulation results of Experiment 1 are shown in Figure 3.



```
>> T1 = [0.1 0 0;0 0.2 0;0 0 0.3]

T1 =

    0.1000         0         0
         0    0.2000         0
         0         0    0.3000
```

```
>> [K1,Kg1] = Calc_Mat(SS1,T1)

K1 =

   -0.2000    1.5000   -0.5000
   -0.1000   -0.7000         0
    0.1000   -0.2333   -0.1333


Kg1 =

    1.5000         0         0
         0    4.0000         0
         0         0    0.0864
```

***Figure 2**. Correction factors*

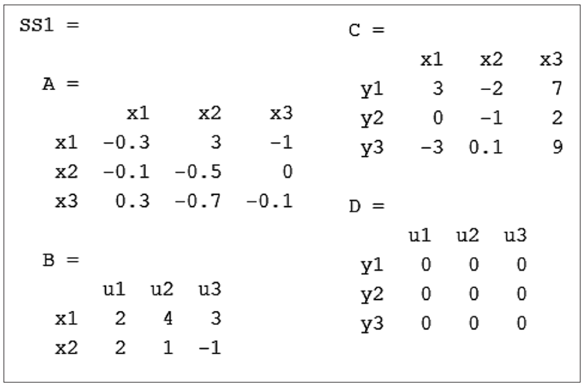


a) *b)* *c)*

***Figure 3**. Simulation results of Experiment 2: a) transients in the initial multi-connected digital system by state and output; b) transients with a feedback controller by state; c) transients using correction coefficients*

Experiment 2. The model of the initial system (Fig. 4).

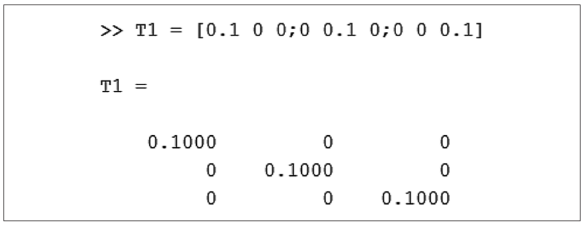
Selected desired roots (Fig. 5).



```
SS1 =

  A =
         x1    x2    x3
   x1  -0.3     3    -1
   x2  -0.1  -0.5     0
   x3   0.3  -0.7  -0.1

  B =
       u1  u2  u3
   x1   2   4   3
   x2   2   1  -1
```

```
C =
       x1   x2   x3
   y1   3   -2    7
   y2   0   -1    2
   y3  -3  0.1    9

D =
      u1  u2  u3
   y1   0   0   0
   y2   0   0   0
   y3   0   0   0
```

***Figure 4**. Model of the source system*

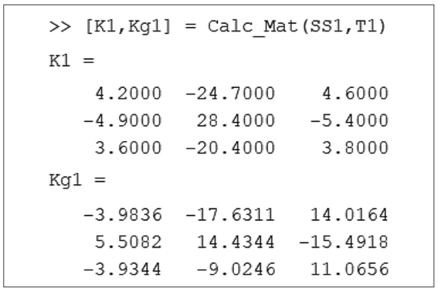


```
>> T1 = [0.1 0 0;0 0.1 0;0 0 0.1]

T1 =

    0.1000         0         0
         0    0.1000         0
         0         0    0.1000
```

***Figure 5.** Selected roots*

Calculated matrices of the regulator and correction coefficients (Figure 6).

The simulation results of Experiment 2 are shown in Figure 7.



```
>> [K1,Kg1] = Calc_Mat(SS1,T1)
K1 =
    4.2000  -24.7000    4.6000
   -4.9000   28.4000   -5.4000
    3.6000  -20.4000    3.8000
Kg1 =
   -3.9836  -17.6311   14.0164
    5.5082   14.4344  -15.4918
   -3.9344   -9.0246   11.0656
```

Figure 6. *Results of calculations of the matrix of correction coefficients*

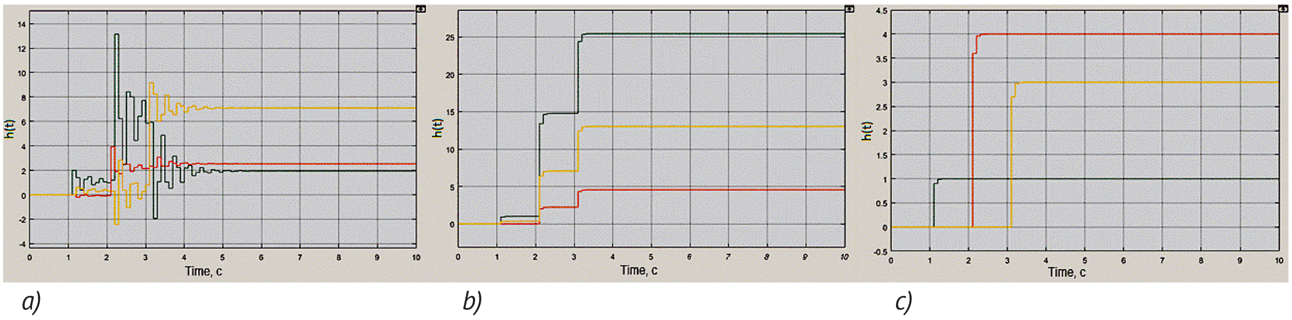
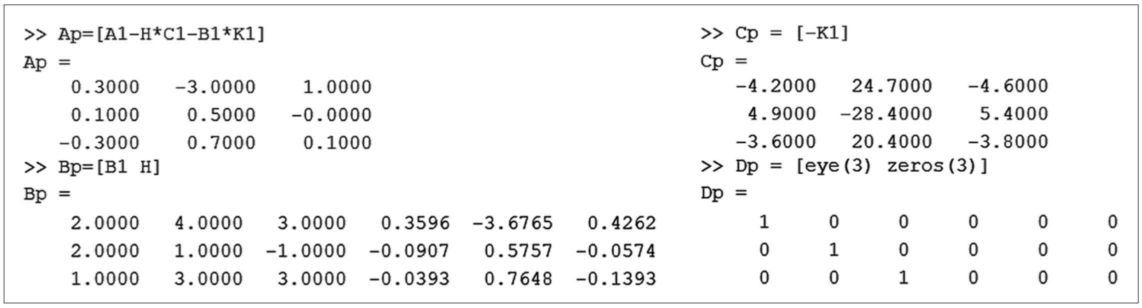


Figure 7*. Simulation results of Experiment 2: a) transients in the initial multi-connected digital system by state and output; b) transients with a feedback controller by state; c) transients using correction coefficients*

Experiment 3.

Calculation of the joint regulator is the final step of the proposed methodology (Fig. 8).

Results of the synthesis of a multi-connected digital system that provides high precision control of various technological processes (Fig. 9).



```
>> Ap=[A1-H*C1-B1*K1]
Ap =
    0.3000   -3.0000    1.0000
    0.1000    0.5000   -0.0000
   -0.3000    0.7000    0.1000
>> Bp=[B1 H]
Bp =
    2.0000    4.0000    3.0000    0.3596   -3.6765    0.4262
    2.0000    1.0000   -1.0000   -0.0907    0.5757   -0.0574
    1.0000    3.0000    3.0000   -0.0393    0.7648   -0.1393
```

```
>> Cp = [-K1]
Cp =
   -4.2000   24.7000   -4.6000
    4.9000  -28.4000    5.4000
   -3.6000   20.4000   -3.8000
>> Dp = [eye(3) zeros(3)]
Dp =
     1     0     0     0     0     0
     0     1     0     0     0     0
     0     0     1     0     0     0
```

Figure 8*. Calculation of the joint regulator*

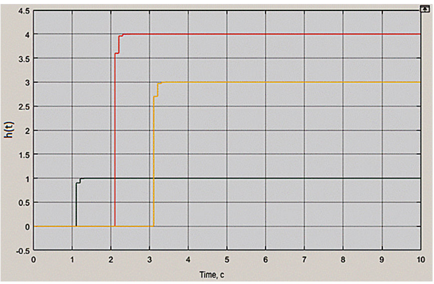


Figure 8*. Transients in a system with an integrated controller providing high control accuracy*

The conducted experiments suggest the following conclusions:

– practical application of the methodology to determine the state feedback matrix makes it possible to eliminate completely the mutual influence of channels;

– derivation of algebraic formulas for determining correction coefficients to ensure zero stable control error contributes to obtaining a qualitative solution to the problem of designating the desired roots of the characteristic equation;

– practical implementation of the decomposition of a digital system with the designation of a vector-matrix model of the regulator is necessary to combine the functions of regulation and supervision.

Described aspects of method of synthesis of multi-connected digital systems are necessary to understand the essence of the functioning of the system, which provides an increase in control accuracy using the feedback method. At the same time, M. Haidekker (2020) in his research concluded that a joint regulator that ensures high quality parameters of system management characterizes the most stable system.

To date, the issues of designing a multi-connected digital control system should be considered one of the key issues in assessing the degree of importance of the problems of theory of automated process control. Cross-links in the model of the desired object significantly reduce its overall resistance to environmental fluctuations, as well as reduce the quality indicators of the entire system when implementing tasks related to stabilization of main technological processes. According to K. Wu (2016), in systems characterized by presence of finite delays, such problems become significantly more complicated, since presence of any uncertainties in setting coefficients or key parameters of the model and delay values (up to 50% of the nominal value) negatively affects sensitivity of the output to uncertainty, causing its decrease, which is associated with a finite margin of stability.

Following key principles of the theory of multi-connected systems, the higher the dynamic quality of a system with many interrelated parameters, the more accurate the incoming signal is processed inside the system, directly in all control channels. At the same time, each output value, and the degree of influence of control in each channel on all other output variables, interaction of which is due to existing mutual connections, must be considered. The presented scientific concept by A. Azar (2021) allows considering an autonomous type system as optimal, where mutual, direct influence of individual, private channels should be completely excluded by compensating for cross-links. This approach is most often used in practice when implementing projects on automation of technological processes, giving predictably high practical results.

The problems of matrix mode distribution have already been sufficiently covered in the scientific literature (Giraud & Giraud-Audine, 2019; Van Horssen, 2020) and have a qualitative base of algorithmic support. At the same time, it should be considered that the formalism of the spatial state, which holds the algebraic characteristics of matrix-vector components, model representations regarding the technologies of building control processes, has not fully exhausted all the possibilities of its expression. Modern technologies of discharge-pulse treatment of surfaces of products and materials involve the use of an electric explosion as the main source of dosed exposure to a given concentration in specific local zones, with high values of specific electrical parameters. According to A. Lenzen & M. Vollmering (2020), the high-voltage discharge channel directly in the liquid layers converts the energy of the charged field of a capacitor bank into mechanical work, which is a clear illustration of the elementary interaction of individual elements of multi-connected digital systems. The main advantages of technological impact of this kind should be considered the prospects of achieving a prominent level of specific energy indicators, in the context of ensuring high controllability of this process. It is quite possible to realize the main advantage in the case of a comprehensive study of the features of an electric explosion, as well as the formalization of this phenomenon, which involves the construction of a mathematical model for the synthesis of automated process control systems.

Stable regulation of the electric explosion process, as one of the processes of expressing various aspects of the functioning of a multidimensional digital system, implies the need to ensure effective control of the discharge mode, due to the stochasticity of the processes of creating channels of this discharge, uncertainties in the parameters of the external environment, as well as processing objects and disturbing influences that arise at various technological levels of processing. K. Vamvoudakis & S. Jagannathan (2016) convinced that the process of technological processing of the results is associated with some uncertainty caused by the presence of discharges on the contour of the object being processed, when the electrode moves directly over the surface being processed. In turn, C. Mollay *et al*. (2021) considered that its unpredictable relief is explained by the destruction of molding mixtures during the purification of castings, as well as random changes in the shape parameters of products.

The process of growing crystals from melts involves moving the hot melt into a cold zone with strictly defined thermal conditions created and supported by a control system. According to D.-K. Kim & D.-H. Kang (2021), this system is designed to provide a given parameter of the crystallization rate, the shape of the front itself, as well as the axial and radial temperature gradient directly near this front. The currently accepted parameter for the accuracy of controlling the temperature characteristic of the vicinity of the crystallization front is 0.1°C over the entire crystal growth cycle. However, W. Fang *et al*. (2021) noted that the process of growing crystals by traditional methods does not always allow for compliance with such management quality standards. Among main reasons, the difficulties in measuring the internal zone of the crystallization temperature by a direct method should be noted, as well as the inability to adjust the temperature measurement by the capabilities of the control system. The height of the crystal growth unit and the features of its design do not allow the placement of temperature sensors and control system heaters only at a certain distance from the object. The result is the need to figure out the best program for controlling changes in temperature regime at some distance from crystal growth zone, which can ensure creation of an optimal temperature regime in the zone itself. This example clearly proves the difficulties of improving control accuracy in multi-connected digital systems with a considerable number of initial parameters.

High-quality development, design and implementation of effective digital control system to monitor the quality of implementation of the calendar of modern production, characterized by the presence of multiple structural relations in management and departmental levels is of great importance from the point of view of the ability to monitor quickly all the structural changes of the production system and to make timely adjustments to the schedule of production (Van Horssen *et al.*, 2020). The development should be carried out in strict accordance with the requirements imposed on the system, with the disclosure of its main functions and the study of the main information flows between the divisions of the production system.

In general, the essence of the strategic task of drawing up a high-quality schedule for the functioning of a multi-connected digital management system for modern production is to find the main types of products produced by a given, specific enterprise, with a sequence of operations for their release per unit of working time. Improving the accuracy of the control of the main parameters of such a system result in a long-term improvement in the quality of manufactured products and an increase in the manufacturability of the production operations performed, which has a positive effect on the production culture. Each part or product must be provided with a technological description of the manufacturing process, per the available production capabilities. Also, a mandatory aspect in this context is the accounting of material resources necessary to perform production tasks in each case (Kim & Kang, 2021).

Management objects in various spheres of modern industry and economy are characterized by the presence of non-standard dynamic characteristics that tend to deteriorate over time due to technological wear of equipment and loss of quality of production operations performed. The quality of the indicators of the technological process of production is significantly reduced, which causes the urgent need to reconfigure the control system and automation of the workflow. Improving control accuracy in multi-connected digital systems of the production process requires considering the nature of the relationships between all objects included in the system and acting as an integral part of it, which implies the need to consider this factor when designing a production management system (Bhogaraju *et al.*, 2021). According to M. Convertino *et al.* (2019), adaptive systems, made using the methods of automatic identification of parameters, to improve the fundamental principles of invariant, autonomous control, do not make it possible to achieve a qualitative improvement of the control system throughout the technological cycle of the operational use of equipment, which negatively affects the stability of the entire production management system as a whole. This fact is explained by the high degree of sensitivity of autonomous, invariant systems even to weakly expressed changes in the dynamics of objects. The implementation of planned identification for objects characterized by the presence of many connections, which represent the foundation for managing the activities of these objects, is a complex technological problem that has not been fully solved to date.

C. De Souza *et al.* (2021) believed that the main tasks of digitalization of the production schedule formation are determined by the need to increase the rhythm of work processes, as well as reduce the likelihood of occurrence and development of emergency situations associated with equipment failures and forced downtime, and interruptions of the production process. The program for the implementation of such tasks should be embedded in the digital management system of production activities. According to W. Fang *et al.* (2021), this indicates an urgent need to develop a management system with the possibilities of optimization of the structure of the technological production process, adapting to changes in production conditions and evaluation of the correctness of the operation and the desired accuracy of models for capacity planning of production equipment, depending on the implemented operating conditions, and also extremely minimized costs in the implementation directly into the production process.

The digital management system for the activities of a manufacturing enterprise is the main core of the management system for the activities of production systems of various scales and classes. Improving the accuracy of control of processes occurring in multi-connected digital production systems allows achieving high-quality integration into a single whole of automated technological preparation of the production process, including calendar planning and centralized quality control of processed products and parts. The main parameters of multi-connected digital systems, distributed directly in management and practical operation, as a rule, are subject to significant structural changes, if the multiple connection of the bulk of the operated technological facilities is observed (Medus *et al.*, 2021). The development of control options for such objects involves a consistent change in the basic settings of the control system over time. The reconfiguration process involves the mandatory clarification of information on the parameters of a specific object model, which implies the need for mandatory identification, which is not always realistic in a situation with multi-connected objects.

When introducing the principles of robust management of objects that are integral elements of the system, it allows guaranteeing the preservation of proper quality parameters and stability throughout the entire period of technological management. At the same time, the selection and justification of the main criteria for the management of a multi-connected digital system and the optimization of the entire system management complex should be carried out according to pre-defined quality parameters of in-system management, which is essential from the point of view of the subsequent functioning of a multi-connected digital system.

When using methods of synthesis of digital systems with feedback, it is necessary to find the desired characteristic polynomial by state. For systems with one input and output, one can use the Check Step Response Characteristics block of the Simulink toolkit of the MatLab software environment, which simplifies the task of finding the desired roots of the characteristic equation and allows determining them in a short period of time and a method for selecting the desired characteristic polynomial using an analytical formula derived in the literature (Repnikova, 2017). But for control systems with multiple connections, the problem of finding the desired polynomial stays. The derived formula (20) for finding the correction coefficients of the proposed method allows to solve this problem by arbitrarily selecting the desired roots according to certain instructions.

For diagonal matrices B and C, the desired roots for the characteristic equation are chosen arbitrary, for non-diagonal ones, the desired roots are arbitrary, but must be multiples. At the same time, the only limitation in the use of the proposed method are the types of matrices – they must be square, although it should be noted that during the experimental studies, the task was set to test the proposed method for irregular matrices. Curiously, for some cases of digital systems, the desired results were obtained. But it was not possible to generalize the proposals for such matrices today. This is an important scientific experience for subsequent research and publications.

CONCLUSIONS

In this scientific paper, an improved method of synthesis of multi-connected digital systems is proposed, which provides an increase in control accuracy using the feedback method by state and is implemented by:

– using the method of determining the feedback matrix by state, which made it possible to exclude the mutual influence of one control channel on another;

– derivation of an analytical formula to determine the correction coefficients to ensure zero stable control error, which solves the problem of determining the desired roots of the characteristic equation;

– performing the decomposition of a digital system with the definition of a vector-matrix model of a regulator combining the functions of regulation and supervision.

In the future, the task is to expand further the method of digital control systems with non-square matrices. It is expected to create an analytical concept for finding the dependence of the ratio of the dimensions of matrices B and C on the choice of the desired roots of characteristic equations. Further expansion of the method of digital control systems based on the practical application of non-square matrix contributes to improving the accuracy of the tasks being solved and is extremely important from the point of view of the prospects to ensure high control accuracy in digital systems characterized by multiple connections.

In general, the theoretical and practical results obtained during this study can be used in the future as the best methodological framework for further research on improving control accuracy in multi-connected digital systems. This is essential for high quality debugging of the functioning of modern high-tech devices.

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Підвищення точності керування в багатозв'язних цифрових системах

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Анотація. Представлене наукове дослідження є актуальним, оскільки в даний час необхідно розробляти та впроваджувати сучасні системи керування технологічними процесами. Це дозволяє підвищити точність керування в багатозв'язних цифрових системах, математичні моделі яких побудовані на платформі методу простору станів. Метою даного дослідження є розробка нового методу підвищення точності керування в багатозв'язних цифрових системах. Методологічна основа даного дослідження, визначена безпосередньо для якісного вирішення поставленої задачі, включала аналітичні вирази, які функціонально не тільки усувають вплив кожного стану та керування на інші, але й забезпечують високу точність процесів керування. В даному науковому дослідженні отримані результати, що становлять методичний підхід до синтезу векторно-матричних моделей регуляторів з використанням зворотного зв'язку за станом. Сформовано векторно-матричну модель регулятора, що поєднує функцію контролю та управління зі зворотним зв'язком за станом. З використанням обчислювальних можливостей математичного апарату, прийнятого в дослідженні, розраховано матриці регуляторів системи та коефіцієнти корекції. Сформульовані висновки зачіпають різні аспекти практичного застосування методу визначення матриці зворотного зв'язку за станом, виведення аналітичної формули для визначення коефіцієнтів корекції для забезпечення нульової помилки усталеного керування, а також виконання декомпозиції цифрової системи з визначенням векторно-матричної моделі регулятора, що поєднує функції регулювання та контролю. Матеріали та методи роботи повністю відповідають заявленій тематиці і можуть слугувати якісною методологічною основою для наступних досліджень у цьому напрямі

Ключові слова: метод простору станів, якість процесів управління, векторно-матричні моделі, зворотні зв'язки