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Посилення однієї теореми про Коксетер-Евклідовий тип головних частково впорядкованих множин

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Strengthening of a theorem on Coxeter–Euclidean type of principal partyally ordered sets

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Квадратичні форми Тітса, які відіграють важливу роль у сучасній математиці, вперше ввів П. Габріель для скінченних сагайдаків. Він довів, що зв'язні сагайдаки з додатною формою Тітса збігаються з сагайдаками Динкіна. Ця квадратична форма природно узагальнюється на ч. в. множини. Ч. в. множини з додатною квадратичною формою Тітса (аналоги діаграм Динкіна) були класифіковані авторами разом з P -критичними ч. в. множинами (найменшими ч. в. множинами з не додатною квадратичною формою Тітса). Квадратична форма Тітса будь-якої P-критичної ч. в. множини є невід'ємною і коранг її симетричної матриці дорівнює 1. У цій роботі вивчаються квадратичні форми всіх ч. в. множин, які задовольняють ці дві властивості; називаються вони головними. Зокрема, для них розв'язана поставлена в 2014 р. проблема про відповідні евклідові діаграми.

Ключові слова: додатна та невід'ємна квадратична форма, квадратична форма Тітса, P -критична ч. в. множина, головна ч. в. множина, діаграма Динкіна, евклідова діаграма

Among the quadratic forms, playing an important role in modern mathematics, the Tits quadratic forms should be distinguished. Such quadratic forms were first introduced by P. Gabriel for any quiver in connection with the study of representations of quivers (also introduced by him). P. Gabriel proved that the connected quivers with positive Tits form coincide with the Dynkin quivers. This quadratic form is naturally generalized to a poset. The posets with positive quadratic Tits form (analogs of the Dynkin diagrams) were classified by the authors together with the P-critical posets (the smallest posets with non-positive quadratic Tits form). The quadratic Tits form of a P-critical poset is non-negative and corank of its symmetric matrix is 1. In this paper we study all posets with such two properties, which are called principal, related to equivalence of their quadratic Tits forms and those of Euclidean diagrams. In particular, one problem posted in 2014 is solved.

Key Words: positive and non-negative quadratic form, quadratic Tits form, P-critical poset, principal poset, Dynkin diagram, Euclidean diagram

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1 Introduction

In 1972 P. Gabriel [1] introduced an integer quadratic form q_Q for any finite quiver (directed graph) Q, called by him the *quadratic Tits form of the quiver* Q. If Q_0 and Q_1 denote the sets of vertices and arrows of Q, respectively, then the quadratic form q_Q is defined by the following equality:

$$q_Q = q_Q(z) := \sum_{i \in Q_0} z_i^2 - \sum_{i \to j} z_i z_j,$$

where $i \to j$ runs through the set Q_1. Note that, by definition, the quadratic Tits form q_G of an undirected graph (for simplicity, a graph) G is that of a quiver G', which is obtained with the help of selecting an orientation ε on the edges of G (up to the renumbering of the variables, the quadratic form q_G does not depend on choice of ε).

The above quadratic form was introduced by P. Gabriel in connection with the study of representations of quivers (also introduced by him). He proved that a connected quiver is of finite representation type over a field (i.e. has, up to equivalence, only finitely many indecomposable representations) if and only if it is a Dynkin quiver, i.e. the corresponding graph is a (simply faced) Dynkin diagram (see Section 5).

If we talk directly about quadratic forms, from the results of [1] it follows that the connected quivers with positive quadratic Tits form coincide with the Dynkin quivers ("positive" means "positive definite").

The above quadratic form q_Q is naturally generalized (in an ideal sense) to a finite partially ordered set (poset) $S \not\ni 0$:

$$q_S = q_S(z) := z_0^2 + \sum_{i \in S} z_i^2 + \sum_{i<j, i,j \in S} z_i z_j - z_0 \sum_{i \in S} z_i$$

(for the first time it was written in [2] in connection with the study of posets of finite representation type). The quadratic form q_S is called the *quadratic Tits form of the poset S*. The problem of classifying all the posets with positive quadratic Tits form were solved by the authors in [3]. These posets are analogs of the Dynkin diagrams.

In [3] the authors also introduced the notion of P-critical poset and classified such posets up to isomorphism (see the list of posets in Section 6). Namely, a poset S is said to be *P-critical* if its quadratic Tits form is not positive, but the Tits form of any proper subposet of S is positive. So the quadratic Tits form $q_S(z)$ is positive if and only if the poset S does not contain (as a subposet) a P-critical one.

Below by "non-negative quadratic form" we meant "non-negative definite quadratic form" (i.e. the form takes only non-negative values).

Theorem 1. *Let S be a P-critical poset. Then*

(1) *the quadratic Tits form $q_S(z)$ is non-negative;*

(2) $\operatorname{Ker} q_S(z) := \{t \in \mathbb{Z}^{|S|+1} \mid q_S(t) = 0\}$ *is an infinite cyclic group, i.e.* $\operatorname{Ker} q_S(z) = t'\mathbb{Z}$ *for some* $t' \neq 0$ *(equivalently, the symmetric matrix of $q_S(z)$ has corank 1).*

Indeed, by Theorem 2 and Proposition 19 [3] the quadratic Tits form $q_S(z)$ is $\mathbb{Z}$-equivalent to the quadratic Tits form of some critical Kleiner's poset; and the theorem it follows from the well-known properties of Kleiner's posets (see, e.g., [4]).

Thus, the P-critical posets coincides with the minimal posets (relative to full inclusion) of the set $\mathcal{P}_{12}$ of all posets, satysfying conditions (1) and (2). The posets from $\mathcal{P}_{12}$ are called *principal* [5].

In this paper we study principal posets related to equivalence of their quadratic Tits forms and those of graphs. In particular, one problem posted in [6] is solved.

2 Main result

By [5, Proposition 9], for any principal poset J there exists a (simply faced) Euclidean diagram (in other words, extended Dynkin diagram; see Section 5) $DJ \in \{\widetilde{\mathbb{A}}_s, s \geqslant 3, \widetilde{\mathbb{D}}_n, n \geqslant 4, \widetilde{\mathbb{E}}_6, \widetilde{\mathbb{E}}_7, \widetilde{\mathbb{E}}_8\}$, uniquely determined by J, such that the symmetric matrices of the quadratic Tits forms of J and DJ (called in [5] the symmetric Gram matrices) are $\mathbb{Z}$-congruent. DJ is called *the Coxeter–Euclidean type* of J.

In this paper we prove the following theorem, which is formulated in the term of equivalence of quadratic forms.

Theorem 2. *Let S be a principal poset. Then there exists an Euclidean diagram $E(S)$, which is not a cycle (i.e. $E(S) \neq \widetilde{\mathbb{A}}_s$), such that the quadratic Tits forms $q_S(z)$ and $q_{E(S)}(z)$ are $\mathbb{Z}$-equivalent.*

This theorem strengthens the indicated result from [5]. In combination with the uniqueness $E(S) := DS$ with respect to S, we have a solution of Problem 1.6 [6].

Our proof is general (does not contain specific calculations) and is based on the methods of minimax equivalence of posets and stable equivalence of quadratic forms.

3 Preliminary

3.1. Minimax equivalence of posets. The notiion of *(min, max)-equivalence of posets* was introduced by the first author in [7]. In details the properties of this equivalence were studied in [3].

Since some time we have been used the term *minimax equivalence.*

In this subsection we remember some definitions and results from [3].

Let S be a (finite) poset. For a minimal (respectively, maximal) element a of S, denote by $T = S_a^{\uparrow}$ (respectively, $T = S_a^{\downarrow}$) the following poset: $T = S$ as usual sets, $T \setminus a = S \setminus a$ as posets, the element a is maximal (resp. minimal) in T, and a is comparable with x in T if and only if they are incomparable in S. A poset T is called minimax equivalent to a poset S, if there are posets $S_1, \ldots, S_p$ ($p \geqslant 0$) such that, if we put $S = S_0$ and $T = S_{p+1}$, then, for every $i = 0, 1, \ldots, p$, either $S_{i+1} = (S_i)_{x_i}^{\uparrow}$ or $S_{i+1} = (S_i)_{y_i}^{\downarrow}$.

The notion of minimax equivalence can be naturally continued to the notion of *minimax isomorphism*: posets S and S' are minimax isomorphic if there exists a poset T, which is minimax equivalent to S and isomorphic to S'.

The definition of posets of the form $T = S_a^{\uparrow}$ (respectively, $T = S_a^{\downarrow}$) can be extended to posets of the form $T = S_A^{\uparrow}$ (respectively, $T = S_A^{\downarrow}$), where A is a lower (respectively, an upper) subposet of S, i.e. $x \in A$ whenever $x < y$ (respectively, $x > y$) and $y \in A$. Namely, $T = S_A^{\uparrow}$ (respectively, $T = S_A^{\downarrow}$) is defined as follows: $T = S$ as usual sets, partial orders on A and $S \setminus A$ are the same as before, but comparability and incomparability between elements of $x \in A$ and $y \in S \setminus A$ are interchanged and the new comparability can only be of the form $x > y$ (respectively, $x < y$).

We write $S_{AB}^{\uparrow\uparrow}$ instead of $(S_A^{\uparrow})_B^{\uparrow}$. $S_{AB}^{\uparrow\downarrow}$ instead of $(S_A^{\uparrow})_B^{\downarrow}$, etc. Obviously, $S_{AA}^{\uparrow\downarrow} = S$, $S_{AA}^{\downarrow\uparrow} = S$, $S_A^{\uparrow} = S_{S\setminus A}^{\downarrow}$. In the special case, when $A = \{a\}$ and $B = \{b\}$ are one-element posets, we identify A and B with a and b.

A poset T is called *dual* to a poset S and is denoted by S^{op} if $T = S$ as usual sets and $x < y$ in T if and only if $x > y$ in S. From the above definitions it follows the next equality: $(S_A^{\downarrow})^{\mathrm{op}} = (S^{\mathrm{op}})_{A^{\mathrm{op}}}^{\uparrow}$.

It is easy to show that $S_A^{\uparrow}$ (respectively, $S_A^{\downarrow}$) and S is minimax equivalent. Since $S_B^{\downarrow} = S_{S\setminus B}^{\uparrow}$, it is sufficient to consider only the case, when the subspace A is upper. Let A_1 be the set of all minimal elements of A and (inductively) $A_i, i > 1$, the set of all minimal elements of $A \setminus (\cup_{j=1}^{i-1} A_j)$ (obviously, $\cup_{i=1}^{r} A_i = A$, where r is the largest i such that $A_i \neq \varnothing$); the writing $h(x) = i$ for an element $x \in S$ will be meant that $x \in S_i$. From this notation it follows that if $|A| = m$, than the elements of A can be numerated in such a way, say $a_1, a_2, \ldots, a_m$, that $h(a_1) \leqslant h(a_2) \leqslant \ldots \leqslant h(a_m)$. Consequently, $S_A^{\uparrow} = S_{a_1 a_2 \ldots a_m}^{\uparrow\,\uparrow\,\ldots\,\uparrow}$, i.e. $S_A^{\uparrow}$ is minimax equivalent to S, as claimed.

The main motivation for introducing the notion of minimax equivalence is the following theorem.

Theorem 3. *The Tits quadratic forms of minimax equivalent posets are $\mathbb{Z}$-equivalent.*

The theorem follows from the next proposition.

Proposition 1. *Let S be a poset and let $T = S_A^{\uparrow}$ or $T = S_A^{\downarrow}$. Then $q_S(z) = q_T(z')$, where $z_0' = z_0 - \sum_{a\in A} z_a$, $z_x' = -z_x$ for $x \in A$ and $z_x' = z_x$ for $x \notin A$.*

Corollary 1. *Let S and T be the same as in Proposition 1, and let $x \notin A$. Then the quadratic Tits form of $T \setminus x$ is positive if so is the quadratic Tits form of $S \setminus x$.*

3.2. Stable equivalence of quadratic forms. Let $f(z) = f(z_1, z_2, \ldots, z_n)$ be a quadratic form of n variables over the field $\mathbb{R}$ of real numbers with the symmetric matrix

$$F = M(f) :=$$

$$= \begin{pmatrix} f_{11} & \frac{f_{12}}{2} & \ldots & \frac{f_{1,n-1}}{2} & \frac{f_{1n}}{2} \\ \frac{f_{12}}{2} & f_{22} & \ldots & \frac{f_{2,n-1}}{2} & \frac{f_{2n}}{2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{f_{1,n-1}}{2} & \frac{f_{2,n-1}}{2} & \ldots & f_{n-1,n-1} & \frac{f_{n-1,n}}{2} \\ \frac{f_{1n}}{2} & \frac{f_{2n}}{2} & \ldots & \frac{f_{n-1,n}}{2} & f_{nn} \end{pmatrix}.$$

Then the quadratic form can be written in the following matrix form:

$$f(z) = \sum_{i=1}^{n} f_{ii} z_i^2 + \sum_{i<j} f_{ij} z_i z_j =$$

$$= (z_1, z_2, \ldots, z_n)\, M(f) \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{pmatrix} = zM(f)z^T,$$

where the letter T means matrix transposition.

Note that we follow the paper [9], and the equality $f(z) = zM(f)z^T$ is not entirely traditional (as a final one), since a replacement is usually done, say $Z := (z_1, z_2, \dots, z_n)^T$ (see [8, Ch. 5,§1]), and then $f(z) = Z^T M(f) Z$.

If in the quadratic form $f(z)$ we perfom a linear transformation $z = yA$ with $y = (y_1, y_2, \dots, y_n)$ and an nonsingular $n \times n$ matrix A, then we get the quadratic form

$$\overline{f}(y) = (yA)F(A^T y^T) = y\left(AFA^T\right)y^T.$$

From this it follows, in particular, that $M(\overline{f}) = AM(f)A^T$.

A quadratic form $f(z) = f(z_1, \dots, z_n)$ is said to be *decomposable* if there is a proper subset of $S \subset N := \{1, 2, \dots, n\}$ such that $f_{ij} = 0$ for $i \in S, j \in N \setminus S$ and for $i \in N \setminus S, j \in S$; otherwise, the form is called *indecomposable*.

We now recall some definitions given in [9].

An $n \times n$ matrix A is called *s-stable*, where $s \in \{1, 2, \dots, n\}$, if its s-th column coinsides with the s-th column of the identity $n \times n$ matrix E. A linear nonsingular transformation $z = yA$ (see above) is called *s-stable* if so is the matrix A.

Two quadratic forms $f = f(z)$ and $g = g(y)$ are called *s-stable equivalent* if there exists a nonsingular linear transformation $z = yA$ being s-stable that carries $f(z)$ into $g(y)$. If $f = f(z)$ and $g = g(y)$ are integer quadratic forms, then the term "s-stable $\mathbb{Z}$-equivalent" means that the s-stable matrix A is integer and invertible (as a matrix over $\mathbb{Z}$).

We now consider the general case of unit integer positive quadratic forms:

$$f(z) = f(z_1, \dots, z_n) = \sum_{i=1}^{n} z_i^2 + \sum_{i<j} f_{ij} z_i z_j,$$

where $n \geqslant 1$; from positivity it follows that $f_{ij} \in \{0, 1, -1\}$ for all i, j. The set of all such quadratic forms is denoted by $\mathcal{Z}_n^+$.

Theorem 4 ([9], Theorem 3)**.** *For any indecomposable quadratic form $f = f(z) \in \mathcal{Z}_n^+$ and $s \in \{1, \dots, n\}$ there exists an s-stably $\mathbb{Z}$-equivalent quadratic form which is the Tits quadratic form of a certain Dynkin diagram.*

Note that a similar statement, but without additional restrictions on equivalence, has long been known [10].

4 Proof of Theorem 2

We will call a poset positive (respectively, non-negative) if so is its quadratic Tits form.

Let S be a principal poset of order $n \geqslant 1$.

Then there is a non-zero integer vector $t = (t_i)_{i \in S \cup 0}$ such that $q_S(t) = 0$. Fix $t_d \neq 0$ with $d \in S$ and consider the subposet $S_0 := S \setminus d$, which is positive by the definition of principal poset. Put $A := \{x \in S \,|\, x < d\}$ and $B := \{x \in S \,|\, x > d\}$. Then the poset $S_d := S^{\uparrow\downarrow}_{AB}$ is a poset with "isolated" element d (in the sense that it is incomparable with any other element).

By Theorem 3 the poset S_d is non-negative (and even principal by Proposition 1) and by Corollaries 1 the poset $S_{d0} := S_d \setminus d$ is positive. Therefore, it suffices to prove the theorem for the poset S_d.

We assume that the elements of S are numbered by the numbers $1, 2, \dots, n$ in such a way that $n = d$, and for the partial order relation on S use (to avoid ambiguity) the symbol $\prec$ instead of $<$. Put $M := M[q_{S_d}(z_0, z_1, \dots, z_n)]$ (the symmetric matrix of the quadratic Tits form of S_d) and $N := M[q_{S_{d0}}(z_0, z_1, \dots, z_{n-1})]$ (the symmetric matrix of the Tits quadratic form of S_{d0}); the rows and columns of the both matrices are numbered by $0, 1, \dots$ in a natural manner (in increasing order). Obviously, $M = \left(\begin{array}{c|c} N & v^T \\ \hline v & 1 \end{array}\right)$, where $v = (-\frac{1}{2}, 0, \dots, 0)$.

By Theorem 4 for $S = S_{d0}$ and $s = 0$ there is a 0-stable matrix

$$A = \begin{pmatrix} 1 & A_{12} \\ 0 & A_{22} \end{pmatrix}$$

with A_{22} to be an $(n-1)\times(n-1)$ matrix (then A_{12} is an $1 \times n - 1$ one) such that $ANA^T = M[q_D(z)]$ for some Dynkin diagram D (the vertices of which are numbered by $0, 1, \dots n-1$). Then, for $\overline{A} = \left(\begin{array}{c|c} A & 0 \\ \hline 0 & 1 \end{array}\right)$, we have (taking into account that $vA^T = v$): $\overline{A} M \overline{A}^T = \left(\begin{array}{c|c} M[q_D(z)] & v^T \\ \hline v & 1 \end{array}\right)$, i.e. $\overline{A} M \overline{A}^T$ is the symmetric matrix of the quadratic Tits form of the graph $\overline{D}$ that is obtained from the Dynkin diagram D by adding the single new vertex n and the single new edge $(0, n)$. Obviously, the (connected) graph $\overline{D}$ is a tree (because so is the diagram D).

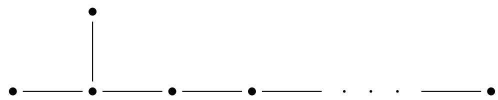
Thus, the quadratic Tits forms $q_S(z)$ of the poset and $q_{\overline{D}}(z)$ of the tree $\overline{D}$ are $\mathbb{Z}$-equivalent. Since $q_{\overline{D}}(z)$ is non-negative and is not positive (because so is $q_S(z)$ with S to be principal), the graph $\overline{D}$ is an extended Dynkin diagram (see [11]).

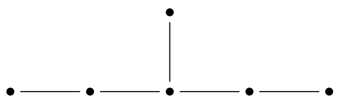
5 Dynkin and Euclidean diagrams

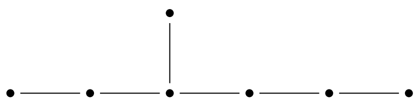
In this section, for reader's convenience, we provide the list of all simply faced Dynkin diagrams — $\mathbb{A}_n$ ($n \geqslant 1$ vertices), $\mathbb{D}_n$ ($n \geqslant 4$ vertices), $\mathbb{E}_6, \mathbb{E}_7, \mathbb{E}_8$ (respectively, 6, 7, 8 vertices), and the list of all simply faced Euclidean (extended Dynkin) diagrams — $\widetilde{\mathbb{A}}_n$ ($n+1 \geqslant 3$ vertices), $\widetilde{\mathbb{D}}_n$ ($n+1 \geqslant 5$ vertices), $\widetilde{\mathbb{E}}_6, \widetilde{\mathbb{E}}_7, \widetilde{\mathbb{E}}_8$ (respectively, 7, 8, 9 vertices).

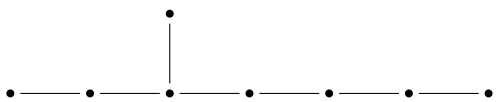
Simply faced Dynkin diagrams

A_n

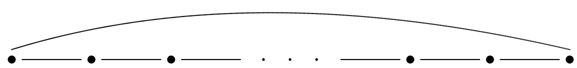
D_n 

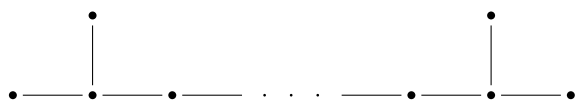
E_6 

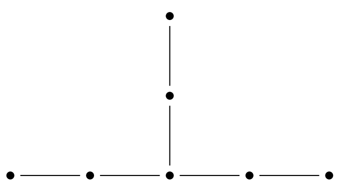
E_7 

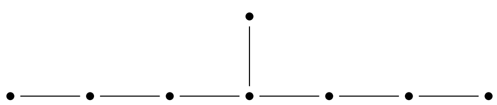
E_8 

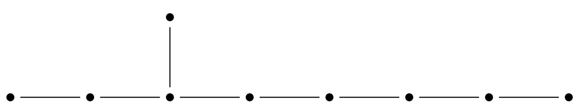
Simply faced Euclidean diagrams

$\widetilde{A}_n$ 

$\widetilde{D}_n$ 

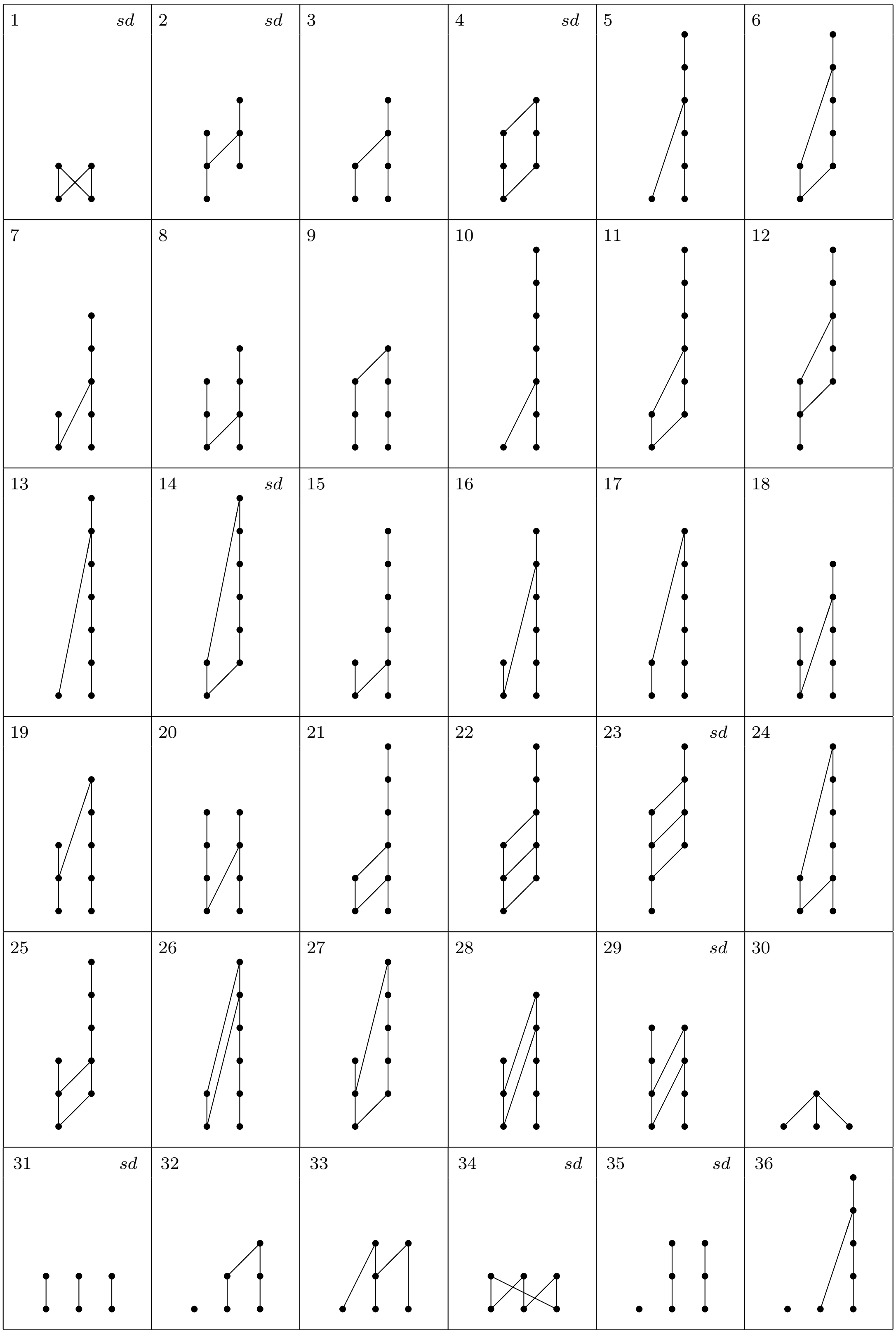
$\widetilde{E}_6$ 

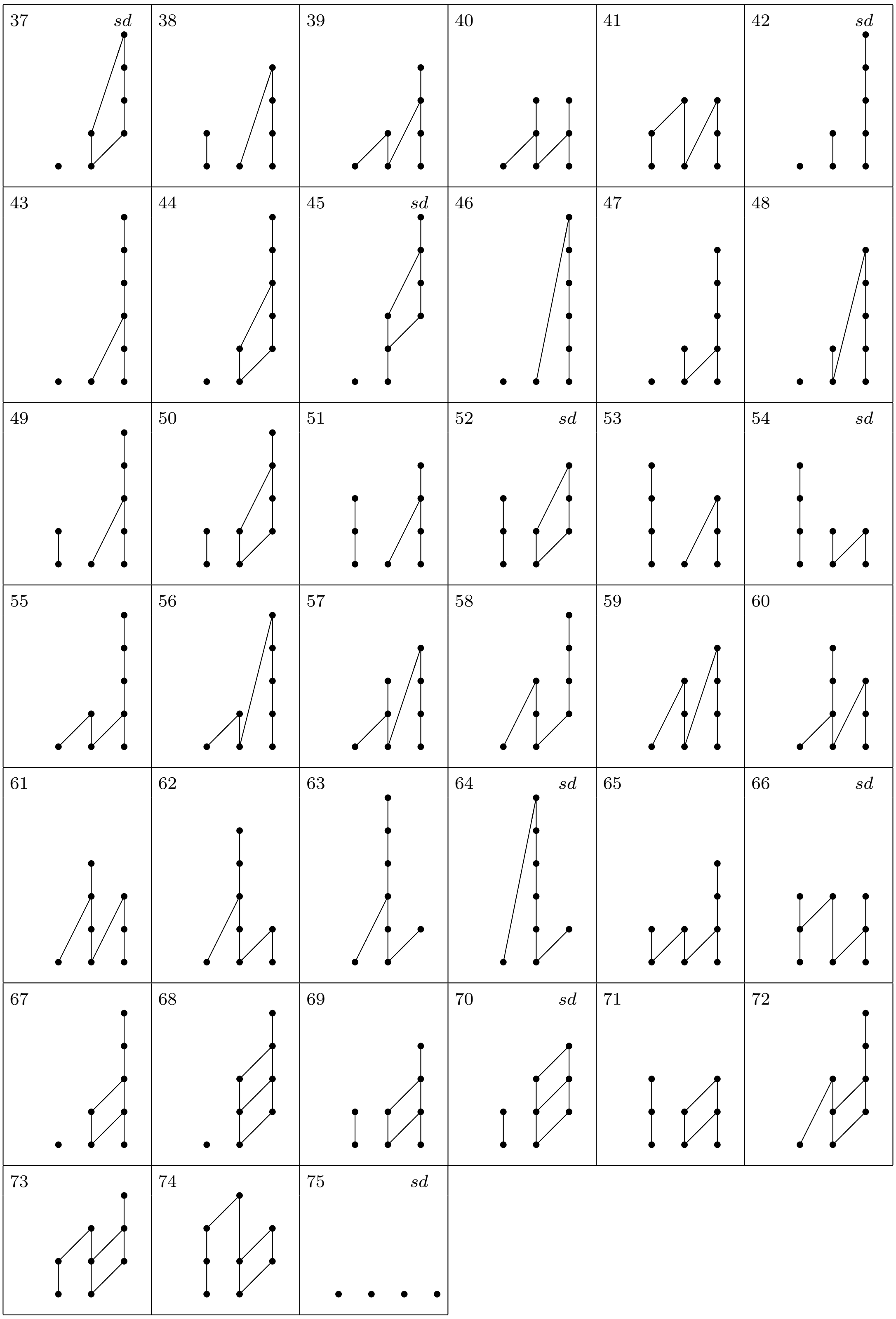
$\widetilde{E}_7$ 

$\widetilde{E}_8$ 

6 The table of the P-critical posets

We follow the paper [3]. The P-critical posets are written up to isomorphism and dyality; their number is 75: $PC_1, PC_2, \ldots, PC_{75}$. Self-dual posets are marked (in the upper right corners) by *sd*. If we add all the posets dual to unmarked ones, we obtain the classification of P-critical posets up to isomorphism; their number is 132: PC_k for $k = 1, 2, \ldots, 75$ and PC_s^{op} for $s \neq 1, 2, 4, 14, 23, 29, 31, 34, 35, 37, 42, 45, 52, 54, 64, 66, 70, 75$.





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