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Modelling plant growth dynamics using the example of paulownia (*Paulownia energy*) seedling growing

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Abstract. The purpose of the study was to develop and verify a nonlinear dynamic model of individual tree growth using the example of seedlings of the fast-growing *Paulownia energy* species. A set of studies on mathematical modelling of physicochemical processes accompanying the growth of seedlings was conducted, including computational and field experiments to assess the adequacy of the proposed model. It has been established that during the growing of seedlings, the rate of phytomass loss through, respiration increases proportionally with the increase in leaf area. During the period of no irrigation, along with the loss of mass due to evaporation of moisture from the soil, a 1.3 time increase in seedling mass was recorded. An increase in the phytomass of the seedling by 63% was established solely due to respiration and the intake of assimilates, and without considering the mass of the pot. The high correlation coefficient ($R^2 = 0.99$) confirmed the high accuracy of the obtained experimental data. Therefore, to maintain stable growth, it is necessary to

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ensure continuous micro-irrigation and active photosynthesis. The developed three-stage nonlinear kinetic model in qualitative terms reliably reflects the process of seedling growth (the correlation coefficient exceeds 90%). This facilitates its use as a basic model for predicting growth rates. It has been confirmed that accurate determination of the kinetic parameters of the process is necessary to ensure quantitative compliance. Based on research of the fast-growing species of paulownia (*Paulownia energy*), a nonlinear dynamic model of tree growth was created. The results obtained provide a reliable opportunity (at the level of $R^2 = 0.99$) to predict the rate of seedlings growing and determine the optimal conditions for their growth

Keywords: paulownia; kinetic model; seedling growth; dynamic phase variables; phytomass increase; integral curves

INTRODUCTION

Contemporary approaches to forest modelling consider numerous factors, including climatic conditions, biological processes, and anthropogenic impacts, which contributes to a more complete picture of tree growth and stand productivity. These approaches rely not only on empirical observations but also on data-driven simulation techniques that integrate remote sensing, machine learning algorithms, and physiological modelling. By incorporating variables such as soil moisture dynamics, temperature fluctuations, light availability, CO_2 concentrations, and disturbance regimes, contemporary forest models enable researchers to assess ecosystem responses under both current and projected environmental scenarios. Furthermore, the inclusion of anthropogenic drivers—such as land use change, forest management practices, and pollution – allows for a more accurate representation of real-world dynamics, enhancing the predictive capacity of models in both natural and managed forest systems.

Researchers M. Blumstein *et al.* (2023) investigated how nonstructural carbohydrate (NSC) reserves in trees vary globally in response to climatic gradients. The study found that temperature and moisture availability are key drivers influencing NSC storage patterns, with significant implications for tree resilience and forest productivity under climate change. T.A.M. Pugh *et al.* (2020) analysed the sources of uncertainty in global forest carbon turnover using a multimodel approach. Their findings highlighted substantial variability stemming from model assumptions about mortality rates, carbon residence times, and climate sensitivity, emphasising the need for harmonised datasets and improved model structures. K. Merganicova *et al.* (2019) focused on modelling forest carbon allocation dynamics under projected climate change conditions. They demonstrated that changes in temperature and precipitation regimes alter allocation patterns between above- and belowground biomass, underscoring the importance of incorporating flexible allocation schemes in dynamic vegetation models.

There are several basic principles for creating mathematical models in biology. One of the main modelling methods is the ordering of balance relations (usually in the form of conservation laws). In dynamic models, balance relations are typically written as systems of differential equations that relate the rates of process flow. This is often done according to Liebig's principle of limiting factors. H.A. Echavarria-Heras *et al.* (2021) in their research argued that the intensity of the output process at each point in time is determined by the minimum component of the input process, that is, the component that prevents others from having their effect. The number of factors influencing the functioning of an ecological system is extremely large, and when creating a model, it is necessary to select the main ones (Mengel, 2019). For example, the development of phytoplankton in seas and lakes is never limited by the carbon content of the water, but is very frequently inhibited by a lack of nitrogen or phosphorus. Therefore, according to A. Cabon & W.R.L. Anderegg (2023), the model must primarily consider the dynamics of nitrogen or phosphorus concentrations, and the concentration of carbon (at least at the first stages of model formulation) should be neglected.

R. Peters *et al.* (2014) determined that such relationships exist between body weight, its surface area, and volume. The use of allometric relationships allows significantly reducing the number of independent model variables. At the same time, dynamic ecophysiological models of whole-tree growth provide an opportunity to combine existing knowledge about physiological and ecophysiological processes and phenomena into a single system. This allows identifying bottlenecks, quantitative discrepancies, and unresolved issues for further ecophysiological research. The decisive factor in dynamic modelling of tree stand growth, according to M.V. Fonti *et al.* (2022), is how much the growth models of an individual tree can be improved and refined. Therefore, the creation of a tree growth model is a central component in most modern forest ecosystem research programmes. Existing tree growth models have considered empirical growth equations in detail. Dynamic modelling of tree growth, which began in the late 1960s, has been addressed in reviews without much separation between tree and stand models. However, it would be more correct to select from the models for comparison those parts (submodels) that describe the same phenomena (Petter *et al.*, 2021).

The purpose of the study was to create and test a nonlinear dynamic model of an individual tree growth using the example of seedlings of the fast-growing species of *Paulownia energy*. To achieve the goal, the task was set to conduct a set of studies on mathematical modelling of physicochemical processes occurring during the growth of seedlings, including computational and full-scale experiments to assess the adequacy of the created model.

MATERIALS AND METHODS

Growth dynamics were investigated on paulownia seedlings (*Paulownia energy*) – 5 pcs – (Fig. 1), which were provided by the company Paulownia energy Ukraine (Lutsk). The samples were in pots during the experiment, the plant height was 120 mm. Before the start of the study, empty pots (containers) and separately, the moist substrate used to grow the experimental seedling were weighed. This helped to obtain the net weight of the samples at each measurement stage by subtracting the previously determined weights of the container and substrate. This procedure ensured the minimisation of the influence of extraneous factors on the weight indicators of the samples under study. The experiment was conducted in laboratory conditions, environmental parameters were as follows: temperature t = 25°C, air humidity $\varphi = 60\%$, controlled ventilation. Light sandy-clay, well-drained soil, pH = 6, was used. Soil moisture was constantly monitored with moderate watering. Illumination of 20/7 for test specimens was provided using a fluorescent lamp. The biomass calculation method was weighing. The duration of the experiment was 43 days. Such pot-based experimental setup allowed maintaining optimal and stable environmental conditions for paulownia growth.

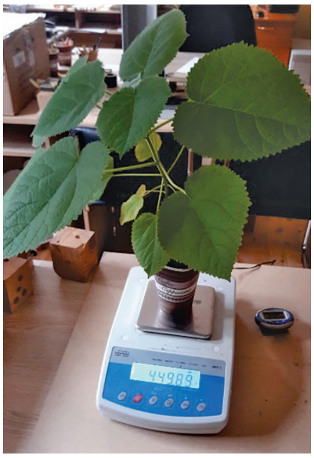

***Figure 1**. Preparation of samples*
***Source:** photo by the authors*

In contrast to open-field conditions where seedlings may be exposed to sharp temperature fluctuations (e.g. heatwaves or frosts), pot cultivation provided thermal buffering that reduced stress. Additionally, controlled irrigation in pots ensured consistent moisture levels, unlike field conditions, where water availability depends on unpredictable rainfall, groundwater fluctuations, and evaporation rates. This controlled setting minimised risks of overwatering or drought and prevents air humidity swings that would otherwise affect plant transpiration. The study was conducted in accordance with the Convention on Biological Diversity (1992).

A computational experiment was conducted using parametric analysis of the constructed mathematical model. The total weight of the plant, which includes the weight of leaves, trunk, roots, and pot with soil, was taken as a phase variable during the modelling. The model described the growth of an individual tree, during which there was a constant change in the total weight of the plant over time. In this case, the change in total weight was determined by the basic kinetic equation, which reflected the law of conservation of mass:

$$dW(t) = dW^{+}(t) - dW^{-}(t), \tag{1}$$

where $dW^{+}(t)$ – increase in total weight due to photosynthesis and assimilates (input), over time $d(t)$, kg; $dW(t) = dW^{+}(t) - dW^{-}(t)$ – reduction in total weight due to plant respiration and soil evaporation over time (losses) $d(t)$, kg.

Considering the physicochemical processes accompanying growth, it can be written:

$$dW^{+}(t) = dW^{+}_{ph}(t) + dW^{+}_{ass}(t), \tag{2}$$

where $dW^{+}_{ph}(t)$ – increase in phytomass due to photosynthesis, kg; $dW^{+}_{ass}(t)$ – increase in phytomass due to the assimilation of substances coming from the soil, kg;

$$dW^{-}(t) = dW^{-}_{br}(t) + dW^{-}_{ll} + dW^{-}_{ev}(t), \tag{3}$$

where $dW^{-}_{br}(t)$ – reduction of phytomass due to respiration (breath), kg; $dW^{-}_{ll}(t)$ – reduction in phytomass due to precipitation (leaflitter), kg; $dW^{-}_{ev}(t)$ – reduction in phytomass due to evaporation of moisture from the pot soil (evaporation), kg.

As an initial hypothesis, it can be accepted that the rate of increase in phytomass and the rate of decrease in phytomass are proportional to the amount of phytomass. This hypothesis is well confirmed in many cases of modelling the dynamics of the living beings' population development or other complex dynamic systems. Following this hypothesis, it is possible to write:

$$\frac{dW^{+}(t)}{dt} = k_1 \cdot W(t), \tag{4}$$

$$\frac{dW^{-}(t)}{dt} = k_2 \cdot W(t), \tag{5}$$

$$\frac{dW(t)}{dt} = k_3 \cdot W^{+}(t) - k_4 \cdot W^{-}(t). \tag{6}$$

In this case, the initial conditions for conducting the experiment will be the following restrictions:

$$t = 0\colon W(t) = W0;\ W^{+}(t) = 0;\ W^{-}(t) = 0, \tag{7}$$

where $k_1(t)$ – rate of increase in phytomass due to photosynthesis and assimilates, kg/s; $k_2(t)$ – rate of phytomass loss due to respiration, precipitation, and soil drying, kg/s; $k_3(t)$ – rate of change in phytomass due to photosynthesis and assimilates, kg/s; $k_4(t)$ – rate of change in phytomass due to respiration, precipitation, and soil drying, kg/s.

In the general case, the rates of change of phytomass cannot be constant values. They depend on many factors that affect growth. These are temperature, solar radiation level, humidity, etc. All of these values were constant. However, in the first approximation, in order to obtain a qualitative image of the kinetics of the process and to have the possibility of qualitative analysis of the integral curves, it was necessary to take them as constant. In order to better understand the development trend, it was better to use dimensionless quantities, because they were the most informative and were not tied to reference systems. In order to be able to compare and study the growth kinetics of several seedlings observed simultaneously, which have different initial weights, it was necessary to introduce new dimensionless values of phytomass measurements, namely, dimensionless phytomass:

$$CB(t) = \frac{W(t)}{W0}. \tag{8}$$

Then, considering the initial conditions (7), it is possible to write them in the form:

$$t = 0 : CB(0) = 1;\ CA(t) = 0;\ CC(t) = 0. \tag{9}$$

Nonlinear dynamic model in the form of a system of nonlinear differential equations:

$$\left\{ \begin{array}{c} \frac{dCA(t)}{dt} = k_1(t)CB(t) \\ \frac{dCB(t)}{dt} = k_3(t)\,CA(t) - k_4CC(t) \\ \frac{dCC(t)}{dt} = k_2(t)\,CB(t) \end{array} \right\}, \tag{10}$$

where $CA(t)$ – increase in the total phytomass of the tree due to photosynthesis and assimilates over time; $CB(t)$ – change in the total phytomass of a growing tree over time; $CC(t)$ – decrease in total phytomass due to respiration over time; $k_1(t)$ – rate of increase in phytomass due to photosynthesis; $k_2(t)$ – rate of phytomass reduction due to respiration; $k_3(t)$ – rate of change in phytomass due to photosynthesis; $k_4(t)$ – rate of change in phytomass due to respiration.

Having solved this system of differential equations (10) under the initial conditions (9), the following values of the change in phase variables over time (integral curves) were obtained:

$$CA(t) := (-2)\cdot k_1 \cdot \frac{CB(0)}{2\cdot k_3\cdot k_1 - 2\cdot k_4\cdot k_2}\cdot [(-k_3)\cdot k_1 + k_4\cdot k_2]^{\frac{1}{2}}\cdot sin\left[[(-k_3)\cdot k_1 + k_4\cdot k_2]^{\frac{1}{2}}\cdot t\right], \tag{11}$$

$$CB(t) := CB(0)\cdot cos\left[[(-k_3)\cdot k_1 + k_4\cdot 2]^{\frac{1}{2}}\cdot t\right], \tag{12}$$

$$CC(t) := (-2)\cdot k_2 \cdot \frac{CB\,(0)}{2\cdot k_3\cdot k_1 - 2\cdot k_4\cdot k_2}\cdot [(-k_3)\cdot k_1 + k_4\cdot k_2]^{\frac{1}{2}}\cdot sin\left[[(-k_3)\cdot k_1 + k_4\cdot k_2]^{\frac{1}{2}}\cdot t\right]. \tag{13}$$

During the field experiment, periodic measurements of the phytomass of seedlings were made along with the pots. In this way, the total weight and its change over time were recorded. Weight measurements were carried out on electronic scales from RADWAG with an accuracy of 0.01 g. In order to determine the amount of seedling respiration at each measurement, parallel measurements of shrinkage were made of the same pot with soil, but without the seedling.

RESULTS AND DISCUSSION

Considering that the system parameters are the rates of change in phytomass, changes in the growth process depend on the ratio of the rates of processes. The values of dynamic phase variables at time are assumed $t = 0 : CB(0) = 1;\ CA(t) = 1;\ CC(t) = 1$. The kinetic diagram also shows the values of the model parameters k_1, k_2, k_3, and k_4, which in this case are variable values and change over time depending on the external and internal conditions of the growth process. Figure 2a shows the kinetic scheme of the model under certain initial conditions $k_1(t) = a$, $k_2(t) = b$, $k_3(t) = c$, $k_4(t) = d$.

The integral curves shown in Figure 2b indicate a constant increase in all phase variable models. Since the arrival of phytomass CA is greater than its consumption CC, the total phytomass CB is also constantly increasing. However, if the model parameters are changed, a corresponding change in the velocities of the phase variables can be seen, that is, the growth rate of the plant. Thus, knowing the values of the parameters from experience, it is possible to predict the growth rate of phytomass. In the initial period, the model shows a growth rate of total phytomass CB less than the rate of its decrease CC. This area is shown in Figure 3.

The results obtained provided that all parameters are equal to each other are presented in Figure 4.

Analysing the integral curves, it can come to the disappointing conclusion that in the case of phytomass growth, there is no CB, since CA = CC. Computational experiment under the condition (condition 2) shown in Figure 5.

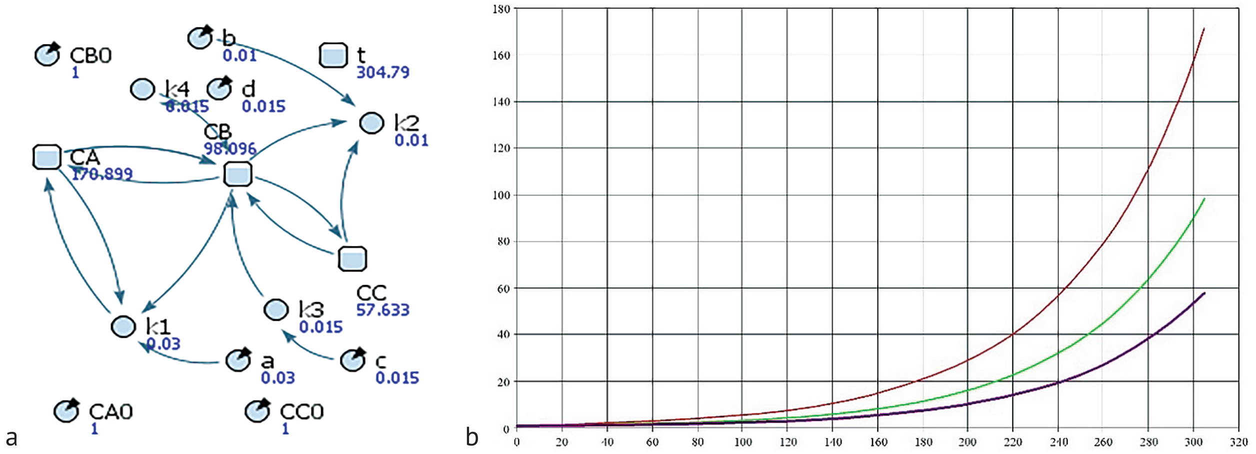

***Figure 2**. Integral curves at the model time point 304.79 s*

***Note:** a – kinetic scheme of the model; b – general curves of changes in dynamic phase variables in time (— $dCA(t)/dt = -k_1 \cdot CB$; — $dCB(t)/dt = k_3 \cdot CA - k_4 \cdot CC$; — $dCC(t) = k_2 \cdot CB$)*

***Source:** compiled by the authors*

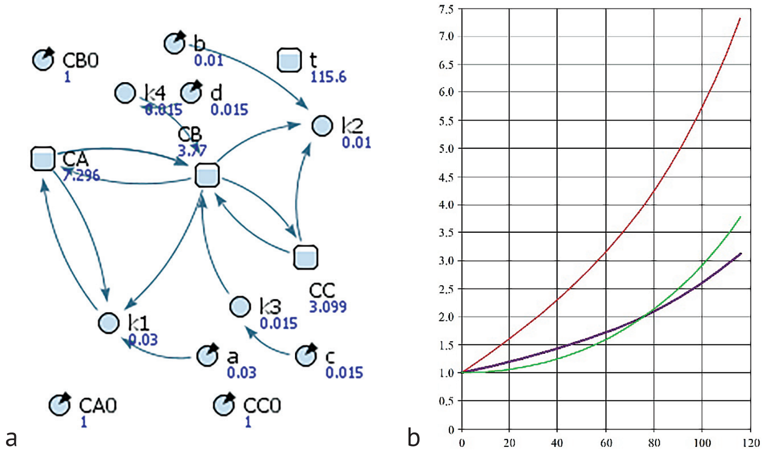

***Figure 3**. Initial section of the computational experiment*

***Note:** a – kinetic scheme of the model; b – curves of changes of dynamic phase variables (— $dCA(t)/dt = -k_1 \cdot CB$; — $dCB(t)/dt = k_3 \cdot CA - k_4 \cdot CC$; — $dCC(t) = k_2 \cdot CB$)*

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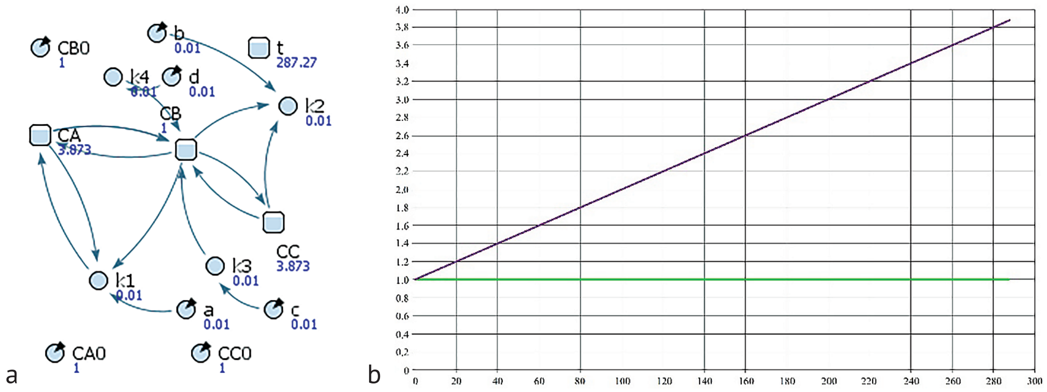

***Figure 4**. Integral curves with the same value of all model parameters*

***Note:** a – kinetic scheme of the model; b – curves of changes of dynamic phase variables (— $dCA(t)/dt = -k_1 \cdot CB$; — $dCB(t)/dt = k_3 \cdot CA - k_4 \cdot CC$; — $dCC(t) = k_2 \cdot CB$)*

***Source:** compiled by the authors*

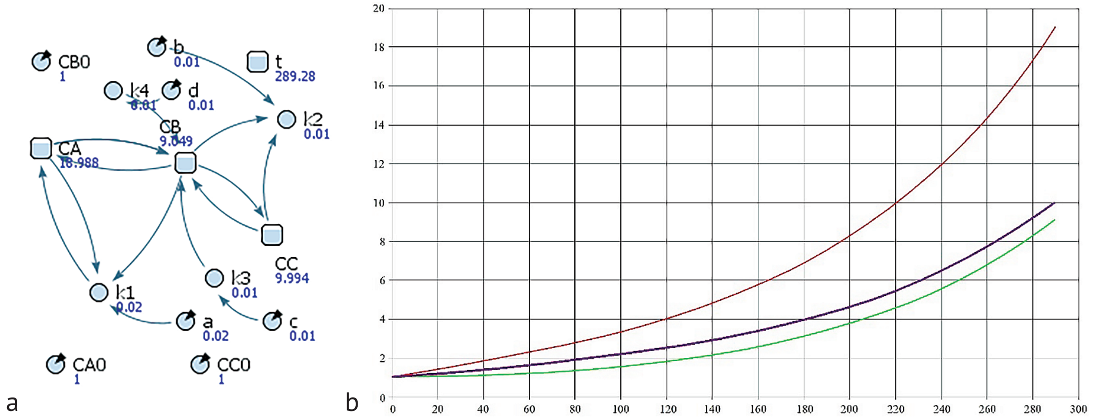

***Figure 5**. Integral curves of phytomass increase (condition 2)*

***Note:** a – kinetic scheme of the model; b – curves of changes in dynamic phase variables (— $dCA(t)/dt=-k_1 \cdot CB$; — $dCB(t)/dt=k_3 \cdot CA-k_4 \cdot CC$; — $dCC(t)=k_2 \cdot CB$)*

***Source:** compiled by the authors*

Analysis of this experiment indicates that an increase in phytomass (*CA*) leads to an increase in total weight (*CB*). The next stage is the study of conditions (condition 3). The results are shown in Figure 6. In this case, due to the high rate of phytomass reduction (*CC*), there is a decrease in total phytomass (CB). Checking the conditions (condition 4) (Fig. 7). A rather paradoxical result – all the curves merged into one integral curve. However, the rates of change of all phase variables are equal to each other (*CA* = *CB* = *CC*). Final computational experiment, checking the conditions (condition 5) (Fig. 8).

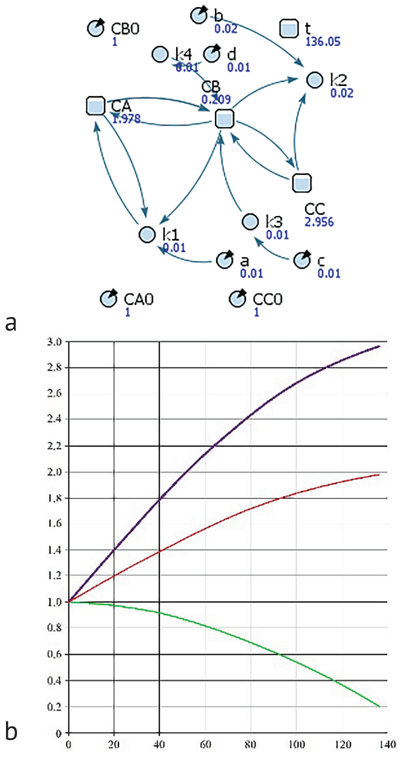

***Figure 6**. Integral curves of phytomass increase (condition 3)*

***Note:** a – kinetic scheme of the model; b – curves of changes in dynamic phase variables (— $dCA(t)/dt=-k_1 \cdot CB$; — $dCB(t)/dt=k_3 \cdot CA-k_4 \cdot CC$; — $dCC(t)=k_2 \cdot CB$)*

***Source:** compiled by the authors*

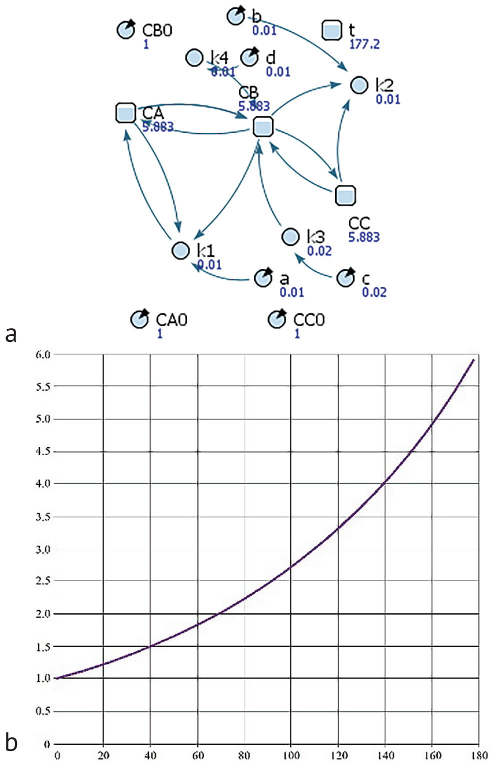

***Figure 7**. Integral curves of phytomass increase (condition 4)*

***Note:** a – kinetic scheme of the model; b – curves of changes in dynamic phase variables (— $dCA(t)/dt=-k_1 \cdot CB$; — $dCB(t)/dt=k_3 \cdot CA-k_4 \cdot CC$; — $dCC(t)=k_2 \cdot CB$)*

***Source:** compiled by the authors*

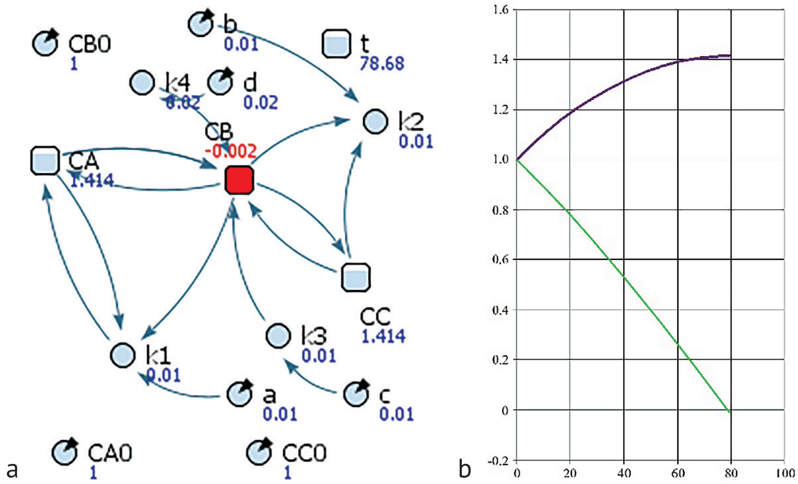

***Figure 8**. Integral curves of phytomass increase (condition 5)*
***Note:** a – kinetic scheme of the model; b – curves of changes in dynamic phase variables (—— $dCA(t)/dt=-k_1\cdot CB$; —— $dCB(t)/dt=k_3\cdot CA-k_4\cdot CC$; —— $dCC(t)=k_2\cdot CB$)*
***Source:** compiled by the authors*

It is quite an expected result, at which the rates of arrival (CA) and expenditure (CC) of phytomass are equal, therefore, there is a decrease in total phytomass (CB). Since photosynthesis rates and phytomass loss due to respiration depend on cyclical changes in weather conditions, dynamic variables also change accordingly. However, the total phytomass of the tree periodically increases. Since this model considers a limited number of variable factors and growth parameters, it may not be entirely adequate. It shows good resilience and the general growth trend of the tree. Therefore, it can be accepted as a basic kinetic model of growth.

Based on the results of the computational experiment, it can be concluded that the proposed model of seedling growth kinetics can be used for qualitative description and optimisation of growth conditions. However, to be confident in a high-quality growth rate forecast, it is necessary to test this model for adequacy, by conducting a full-scale experiment and comparing its results with the modelling results. For this purpose, a full-scale experiment was conducted. Figure 9 presents a listing of part of the results of the measurements, which indicates the dates of the measurements and the weight of the seedlings.

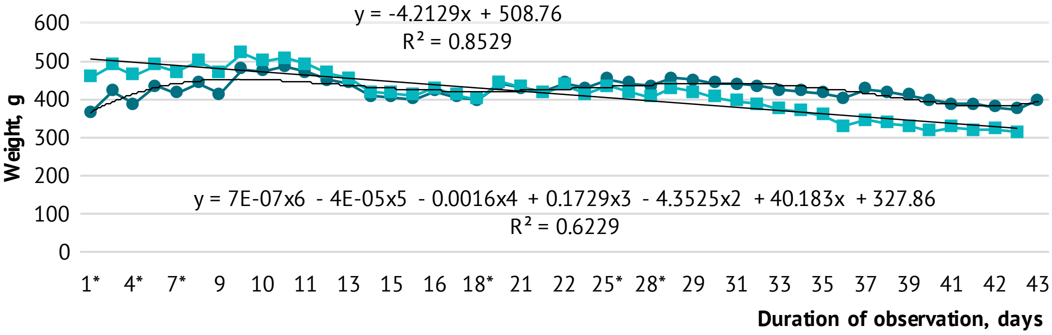

***Figure 9**. Dynamics of changes in the total weight of experimental seedlings*
***Note:** "*" indicates the dates of watering seedlings*
***Source:** compiled by the authors*

Throughout the entire experiment, constant growth of seedlings was visually observed. Simultaneously, the area of leaves also increased. However, as can be seen from the curves presented in Figure 9, the increase in total weight occurs only at the time of irrigation due to additional moisture. When analysing the curve of change in total weight in the initial period, it was noticed that the total weight changes slightly or tends to decrease. Based on these observations, a hypothesis was developed that during the process of phytomass growth due to assimilates and photosynthesis, phytomass simultaneously, decreases due to an increase in respiration intensity. However, the rate of uptake of assimilates from the soil remains practically unchanged.

To test this hypothesis, it is necessary to compare how the total weight of the seedling with the soil and pot changes. Moreover, it is necessary to consider the evaporation losses of the soil itself during the growth process, that is, to exclude this value from the total weight. In order to determine what part of the change in total phytomass is accounted for by soil evaporation, two more pots were prepared with the same soil, but without seedlings. Their watering and weight measurement were carried out in parallel, simultaneously with the seedlings. In order to increase the purity of the experiment by reducing the number of variable factors, watering was not carried out for 6 days for both seedlings and pots without seedlings. The results of measurements of the total weight of pots without watering with seedlings are presented in Figure 10.

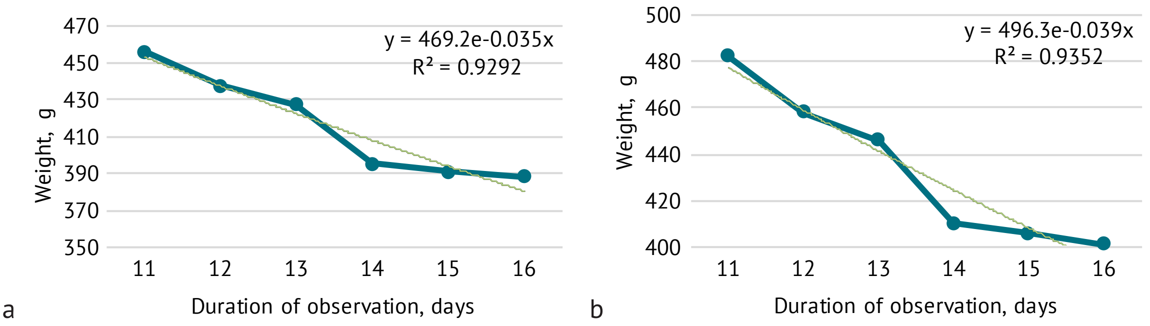

***Figure 10**. Results of measurements of the total weight of samples in the period between watering*
***Note:** a – pot with a seedling; b – pot without a seedling*
***Source:** compiled by the authors*

As can be seen from the results shown in Figure 10, there was a gradual decrease in the weight of pots with and without seedlings. It was noted that the pot without a seedling lost 30 % more weight compared to the seedling during the period without watering. The reliability of the obtained dependence is evidenced by the high correlation coefficient $R^2 = 0.9292$ and $R^2 = 0.9352$, respectively. Figure 11 presents combined graphs of changes in the weight of pots with seedlings and without seedlings in units of dimensionless weight.

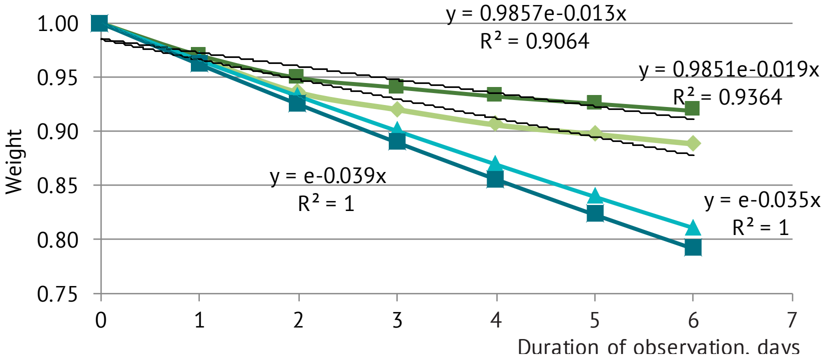

***Figure 11**. Weight changes of pots with seedlings (lower curves) and without seedlings (upper curves) in units of dimensionless weight*
***Source:** compiled by the authors*

In order to exclude the effect of soil drying in pots with seedlings, it is necessary to subtract the values of the change in the total weight of the pots. The results of this operation are presented in Figure 12, which shows the summary graphs of changes in phytomass only due to respiration and the intake of assimilates. In this study, during plant growth modelling, the moment of planting (beginning of observation) means time $x = 0$, that is, growth is not yet observed, and the value of phytomass (variable y) may be close to zero (Fig. 12a). Since the exponential function does not assume zero values (it must always be positive), it is more correct to start observations from $x = 1$, that is, from the first fixed phase of active growth (Fig. 12b).

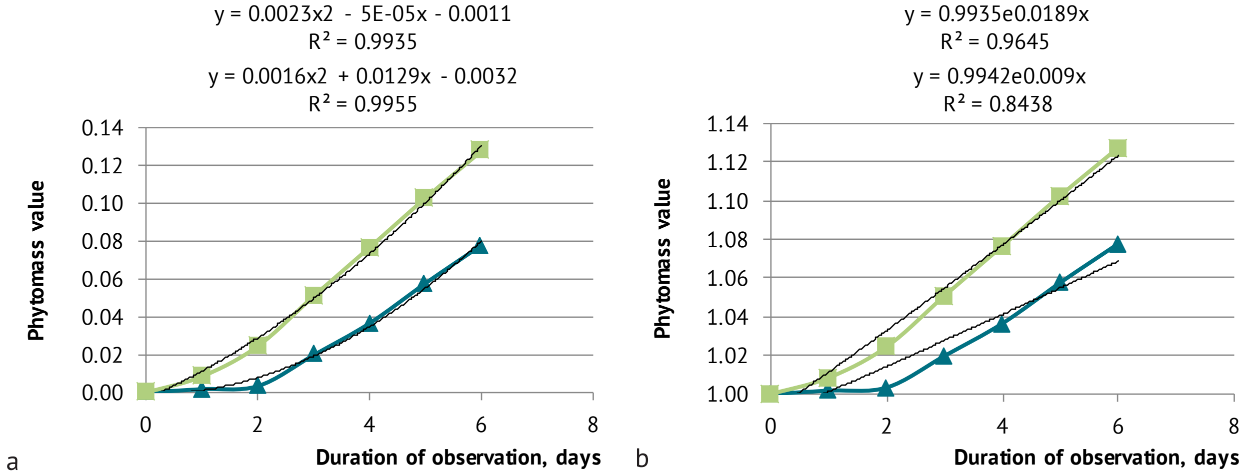

Figure 12. *Changes in phytomass only due to respiration and the intake of assimilate*
Note: *a – under real conditions; b – under conditions of a shifted coordinate system*
Source: *compiled by the authors*

As a result, a correct mathematical model was obtained that corresponds to the biological reality of the seedling growth process and confirms stability during logarithmic analysis and model verification. It is clearly visible that in the first two days the changes are insignificant, and from the third day onwards, a sharp increase in phytomass can be observed. Since the rate of photosynthesis and the loss of phytomass due to respiration depend on cyclical changes in weather conditions (Fig. 7), this directly affects the change in the dynamic parameters of tree growth. After all, temperature fluctuations, light levels, humidity, and other environmental factors determine the intensity of metabolic processes in plants. This is confirmed by the results of H.A. Jabri *et al.* (2021), who noticed the reflection of seasonal and daily environmental variations on biomass productivity. Although fluctuations in metabolic activity can lead to a temporary decrease in organic matter accumulation (Liu *et al.*, 2019), the overall trend in woody phytomass development remains positive. This indicates stable growth of plantations in the long term. However, this model partially considers variable factors and growth parameters, which does not allow it to be completely accurate for all environmental conditions. However, due to its ability to reflect general patterns and adapt to major environmental changes, the model demonstrates high robustness in predicting wood growth trends. This allows it to be used as a basic kinetic model, which can serve as the basis for further improvement and expansion, considering additional biotic and abiotic factors.

Analysing the graphical dependences of changes in the total weight of seedlings (Fig. 9), it can be noted that its increase occurs mainly at the moments of watering, which is conditioned by the saturation of the substrate with additional moisture. This phenomenon is explained by the fact that water is the main component of the total weight of the plant, especially in the initial periods of vegetation, when the root system actively absorbs moisture. In their comparative study of Scots pine growth, V. Lovynska *et al.* (2021) also recorded a significant weight increase of seedlings directly after irrigation, particularly in the early vegetation stages. Their results confirmed that water availability is a decisive factor influencing biomass accumulation and that rapid moisture uptake plays a critical role in shaping the growth dynamics of seedlings in different ecological zones.

A detailed study of the dynamics of weight changes in the initial stages of development indicates relative stability or even a slight decrease in total weight. R. Silvestro *et al.* (2023) consider that this may be due to adaptive processes occurring in plants in response to changing environmental conditions. This is clearly visible at the stage of transition from using internal reserves to actively attracting external resources. However, regardless of minor fluctuations in total weight, a steady trend towards an increase in the phytomass of seedlings was recorded. This was confirmed by the gradual growth of vegetative organs, such as the stem and leaf blade. The intensive increase in leaf area is evidence of the activation of photosynthetic processes, which contributes to the synthesis of organic compounds and, accordingly, the further development of biomass.

Based on the results obtained (Fig. 12), it can be stated that during the growth of seedlings, the respiration intensity continuously increases, obeying quadratic or exponential laws. This is conditioned by acceleration in metabolism in the plant, which is accompanied by an increase in energy expenditure to support life processes. P. Fransson *et al.* (2025) demonstrated that under resource-limited conditions, such as nitrogen and water shortage, plants tend to redistribute energy flow to maintain vital physiological activities, even at the cost of reduced structural biomass accumulation.

Their eco-physiological model confirmed that respiration intensity can dominate over photosynthesis under stress, leading to changes in net carbon gain. However, there is a tendency for the respiration rate to significantly exceed the increase in phytomass caused by the assimilation of organic matter through photosynthesis. An increase in respiration rates causes a relative decrease in the total phytomass of the seedling, since part of the stored assimilates is spent on energy provision for physiological processes, rather than on the formation of new structural components. This highlights the important role of the balance between the level of photosynthetic activity and the intensity of respiration in determining the overall growth and productivity of the plant. R.M. Gifford (2003) argued that many productivity models underestimate the cost of respiration, especially under dynamic environmental conditions, which can lead to overestimations of net primary productivity. He also emphasised the need for integrating realistic respiration dynamics into plant growth models, particularly for fast-growing species. Thus, the absolute growth of a seedling directly depends on the rate of intake of assimilates, the efficiency of the photosynthetic apparatus, and the plant's ability to regulate metabolism in response to external conditions.

Comparative analysis of the breathing curves obtained in the field experiment (Fig. 12) with similar data during computational modelling (Fig. 2b), curve CC(t)) showed their qualitative correspondence. In both cases, an exponential growth of phase variables was recorded, which indicates the reliability and adequacy of the developed mathematical model. Its ability to qualitatively describe the dynamics of physiological changes allows it to be used to predict the growth and development of seedlings under various environmental conditions. This is important for optimising agrotechnical measures and planning forestry operations.

Nowadays, there is significant interest in growing fast-growing tree species with a short life cycle. The main purpose of creating such plantations is to obtain wood fibres and biomass, which are widely used in industry. Moreover, these species have significant potential for the production of finished wood products, which opens up additional prospects for their application. Paulownia is one of the most promising fast-growing species, the area of its plantations in Ukraine is growing rapidly. However, currently, information on the physical and mechanical properties of this wood remains insufficiently studied (Barbu *et al.*, 2023), which causes certain difficulties in determining possible areas of its use in the woodworking industry (Tsapko *et al.*, 2022). It is also important to assess and predict the condition of a product manufactured from this wood under different loading conditions (Kulman *et al.*, 2021; Bkarbu *et al.*, 2023), which will determine its durability. S. Kulman *et al.* (2021) proposed a kinetic approach for long-term strength prediction of wood-based composites and demonstrated the relevance of time-dependent stress factors, which could also be applied to fast-growing hardwoods such as paulownia. M.C. Barbu *et al.* (2023) showed that the mechanical characteristics of paulownia wood significantly varied along the stem height and across radial sections, which required a differentiated approach in selecting material for structural and non-structural applications. C. Huber *et al.* (2023) confirmed this variability and emphasised that paulownia's low density and fast maturity made it suitable for thermal modification and use in lightweight construction, although not all material met mechanical stability standards.

However, despite the successes achieved, both challenges and new opportunities remain for further improvement of the models. The model can be significantly refined by determining the values of the rate constants of the processes occurring, which in general have a thermal activation character according to the Arrhenius type (Kulman *et al.*, 2019). S. Kulman *et al.* (2019) demonstrated that variations in temperature and moisture affected the modulus of elasticity and rupture of particleboards, suggesting the possibility of adapting these parameters for paulownia-based products. In addition, these may be other values, which generally depend on environmental parameters, such as absolute temperature T, humidity W %, solar radiation level R, time of the vegetative or other period t. If the period of daily fluctuations of these quantities is considered, then the rate constants can be represented, for example, in this form . Such integration of dynamic environmental effects into the models allowed for more precise prediction of product behaviour and contributed to enhancing the ecological and practical relevance of forestry strategies. This development process allows not only optimising forestry practices, but also opens up prospects for innovative research in the field of ecology and nature management.

Thus, the conducted modelling enabled the identification of key relationships between the physiological parameters of paulownia seedling growth and environmental conditions. The results confirmed that photosynthetic activity, water availability, and respiration intensity significantly influence the rate of phytomass accumulation. The proposed kinetic model can be applied as a tool for predicting plant growth under variable irrigation and light conditions. Overall, the study provides a foundation for evidence-based approaches to the cultivation of fast-growing tree species, which is highly relevant in the context of sustainable forestry and the bioeconomy.

CONCLUSIONS

During the growth of a seedling, especially at the time of intensive increment in phytomass due to an increase in the area of its leaves, the rate of decrease in phytomass using respiration increases proportionally. Therefore, it is very important to maintain a constant supply of moisture (increase in phytomass) by ensuring continuous micro-irrigation and constant photosynthesis. During the period without watering, along with weight loss due to drying of the soil, an increase in the weight of the seedling by 20 g was recorded. Weight gain was accompanied by an increase in leaf size. If the weight of the pot was subtracted, a 1.63 time increase in phytomass was determined only due to respiration and the intake of assimilates. A correlation coefficient above $R^2 = 0.99$ indicates high accuracy of the experiment.

Comparing weight loss, it was found that the average weight loss rate of pots without seedlings was 0.0115 g/day, and with seedlings it was 2.8 times faster – 0.0325 g/day. That is, the presence of seedlings significantly accelerates weight loss, which is direct evidence of transpiration. This confirms that biological processes are the dominant factor in moisture loss compared to physical evaporation from the soil surface alone. Based on both experiments, the qualitative adequacy of the model was recorded. The exponential growth of phase variables in both cases confirms that the developed model adequately reproduces the overall dynamics of the studied biological processes. The proposed nonlinear kinetic three-stage model in a qualitative form adequately describes the process of seedling growth (correlation coefficient more than 90%) and therefore, can be used as a basic kinetic model for predicting growth rate. For this model to adequately describe the growth process quantitatively, accurate knowledge of the kinetic parameters of the model is necessary. Using this model allows predicting the growth rate of seedlings and creating optimal development conditions for them. Further development of this approach, considering additional factors, will not only optimise forest management methods, but also open up new opportunities for innovative ecological research, ease of monitoring, and effective modelling of forest ecosystem development.

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CONFLICT OF INTEREST

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Моделювання динаміки росту рослин на прикладі зростання саджанців павловнії (*Paulownia energy*)

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Анотація. Метою дослідження була розробка та верифікація нелінійної динамічної моделі росту окремого дерева на прикладі саджанців швидкорослої породи *Paulownia energy*. Проведено комплекс досліджень із математичного моделювання фізико-хімічних процесів, що супроводжують ріст саджанців, включаючи обчислювальні та натурні експерименти з оцінки адекватності запропонованої моделі. Встановлено, що у процесі розвитку саджанців швидкість втрати фітомаси через дихання пропорційно зростає разом зі збільшенням площі листя. У період відсутності поливу, поряд із втратою маси через випаровування вологи з ґрунту, було зафіксовано збільшення маси саджанця в 1,3 рази. Зафіксовано збільшення фітомаси саджанця на 63 % виключно внаслідок дихання та надходження асимілятів та без врахування маси горщика. Високий коефіцієнт кореляції ($R^2 = 0,99$) підтвердив високу точність отриманих експериментальних даних. Тому для підтримання стабільного приросту необхідне забезпечення безперервного мікрополиву та активного фотосинтезу. Розроблена трьохстадійна нелінійна кінетична модель у якісному вираженні достовірно відображає процес зростання саджанців (коефіцієнт кореляції перевищує 90 %). Це сприяє використанню її як базової моделі для прогнозування швидкості росту. Підтверджено, що для забезпечення кількісної відповідності необхідне точне визначення кінетичних параметрів процесу. На основі дослідження швидкорослої породи павловнії (*Paulownia energy*) створено нелінійну динамічну модель росту дерев. Отримані результати забезпечують вірогідну можливість (на рівні $R^2 = 0,99$) прогнозувати темпи розвитку саджанців та визначати оптимальні умови для їхнього росту

Ключові слова: павловнія; кінетична модель; ріст саджанців; динамічні фазові змінні; збільшення фітомаси; інтегральні криві