On the Hasse diagram of P-critical posets

M. V. Styopochkina
Zhytomyr National Agroecological University, Zhytomyr, Ukraine
StMar@ukr.net

For a class of finite posets $\mathcal{X}$ we denote by $\mathrm{VA}(\mathcal{X})$ the set of pairs (s,k) of non-negative integer numbers, such that s and k are respectively the number of vertices and edges of the Hasse diagram $H(X)$ for an $X \in \mathcal{X}$.

We consider the Hasse diagram of posets connected with the Tits quadratic form.

Let S be a poset without an element denoted by 0. The Tits quadratic form of S is by definition the form $q_S : \mathbb{Z}^{S \cup 0} \to \mathbb{Z}$ defined by the equality

$$q_S(z) = z_0^2 + \sum_{i \in S} z_i^2 + \sum_{i<j, i,j \in S} z_i z_j - z_0 \sum_{i \in S} z_i.$$

A poset S is called critical with respect to positivity of the Tits quadratic form or, briefly, P-critical if the Tits form of any its proper subset is positive but the Tits form of S is not positive [1]. The set of all P-critical posets will be denoted by $\mathcal{P}_c$.

Theorem. *VA($\mathcal{P}_c$) consists of the following pairs:* $(4,0)$, $(4,3)$, $(4,4)$, $(6,3)$, $(6,4)$, $(6,5)$, $(6,6)$, $(7,4)$, $(7,5)$, $(7,6)$, $(7,7)$, $(8,5)$, $(8,6)$, $(8,7)$, $(8,8)$, $(8,9)$.

Corollary 1. Let $(s,i),(s,j) \in \mathrm{VA}(\mathcal{P}_c)$ and $i < k < j$. If s is not equal to 4 (the smallest first coordinate for the pairs of $\mathrm{VA}(\mathcal{P}_c)$), then $(s,k) \in \mathrm{VA}(\mathcal{P}_c)$.

Corollary 2. Let $(s,k) \in \mathrm{VA}(\mathcal{P}_c)$. If s is not equal to 9 (the biggest second coordinate for the pairs of $\mathrm{VA}(\mathcal{P}_c)$), then $s \geqslant k$.

These studies were carried out together with V. M. Bondarenko and I. V. Chervyakov.

1. Bondarenko V. M., Stepochkina M. V. (Min, max)-equivalence partially ordered sets and quadratic Tits form. Zb. Pr. Inst. Mat. NAN Ukr., 2005, **2**, no. 3, P. 18–58 (in Russian).