UDC 512.64 + 512.56

ON COEFFICIENTS OF TRANSITIVENESS OF POSETS CRITICAL WITH RESPECT TO THE POSITIVITY OF THE QUADRATIC TITS FORM

Vitaliy Bondarenko, Maryna Styopochkina

Institute of Mathematics of NAN of Ukraine,
Zhytomyr National University of Agriculture and Ecology

vit-bond@imath.kiev.ua, StMar@ukr.net

The quadratic Tits form, introduced by P. Gabriel for a finite quiver, is naturally generalized to a finite poset $0 \notin S$:

$$q_S(z) = z_0^2 + \sum_{i \in S} z_i^2 + \sum_{i<j, i, j \in S} z_i z_j - z_0 \sum_{i \in S} z_i \ .$$

In [1] it is introduced the notion of *P*-critical posets as posets critical with respect to the positivity of the quadratic Tits form. So the *P*-critical posets are analogs of the extended Dynkin diagrams. All such posets are classified in [1].

We study combinatorial properties of *P*-critical posets.

Let S be a finite poset and $S_<^2 := \{(x, y) \mid x, y \in S, x < y\}$. If $(x, y) \in S_<^2$ and there is no z satisfying $x < z < y$, then one says that x and y are neighboring. We put $n_w = n_w(S) := \left|S_<^2\right|$ and denote by $n_e = n_e(S)$ the number of pairs of neighboring elements. On the language of the Hasse diagram $H(S)$ (that represents S in the plane), n_e is equal to the number of all its edges and n_w to the number of all its paths, up to parallelity, going bottom-up (two path is called parallel if they start and terminate at the same vertices). The ratio $k_t = k_t(S)$ of the numbers $n_w - n_e$ and n_w we call the coefficient of transitiveness of S. If $n_w = 0$ (then $n_e = 0$), we assume $k_t = 0$.

An element of a poset T is called nodal, if it is comparable with all elements of T. Obviously, each element of T is nodal iff T is a chain. It follows from the results of [2] that any *P*-critical poset S is uniquely represented in the form $S = S_0^- \cup S_1 \cup S_0^+$ where S_0^-, S_0^+ are chains (maybe empty), S_1 does not contain nodal elements and $S_0^- < S_1 < S_0^+$ ($X < Y$ means that $x < y$ for any $x \in X, y \in Y$). Then $S_0 = S_0^- \cup S_{.0}^+$ is the set of all nodal elements of S.

We calculate the coefficients of transitiveness k_t for each of the 75 critical posets (see the list in [2]). All the coefficients are written up to the second decimal place.

Theorem 1. *The following holds for P-critical posets 1 – 75* :

N	k_t	N	k_t	N	k_t	N	k_t	N	k_t
1	0,00	16	0,61	31	0,00	46	0,63	61	0,42
2	0,50	17	0,61	32	0,33	47	0,60	62	0,53
3	0,55	18	0,53	33	0,29	48	0,50	63	0,65
4	0,45	19	0,53	34	0,00	49	0,57	64	0,59
5	0,67	20	0,50	35	0,33	50	0,50	65	0,42
6	0,61	21	0,68	36	0,58	51	0,45	66	0,36
7	0,57	22	0,64	37	0,50	52	0,36	67	0,61
8	0,50	23	0,64	38	0,38	53	0,40	68	0,56
9	0,50	24	0,64	39	0,40	54	0,33	69	0,46
10	0,73	25	0,64	40	0,40	55	0,56	70	0,38
11	0,69	26	0,58	41	0,25	56	0,46	71	0,30
12	0,69	27	0,58	42	0,55	57	0,42	72	0,53
13	0,70	28	0,50	43	0,68	58	0,50	73	0,43
14	0,65	29	0,47	44	0,63	59	0,36	74	0,38
15	0,67	30	0,00	45	0,63	60	0,46	75	0,00

Theorem 2. *Let S be a P-critical poset. Then the following conditions are equivalent:*

a) $k_t(S) \geq k_t(T)$ *for any P-critical poset T;*

b) $|S_0| \geq |T_0|$ *for any P-critical poset* T *, and* S_0^- *or* S_0^+ *is empty.*

1. *Bondarenko V. M., Styopochkina M. V.* (Min, max)-equivalence of partially ordered sets and the Tits quadratic form // Zb. Pr. Inst. Mat. NAN Ukr. Problems of Analysis and Algebra. – 2005. – **2**, No. 3. – P. 18–58 (in Russian).

ПРО КОЕФІЦІЄНТИ ТРАНЗИТИВНОСТІ Ч. В. МНОЖИН, КРИТИЧНИХ ВІДНОСНО ДОДАТНОСТІ КВАДРАТИЧНОЇ ФОРМИ ТІТСА

Ми вводимо інваріант скінченної ч. в. множини, названий коефіцієнтом транзитивності, і обчислюємо його для всіх P-критичних ч. в. множин.