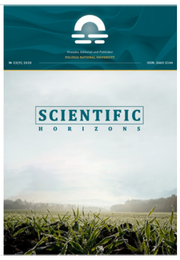
SCIENTIFIC HORIZONS
Journal homepage: https://sciencehorizon.com.ua
Scientific Horizons, 28(8), 30-50
UDC 631.41:633.11
DOI: 10.48077/scihor8.2025.30



Causal relationships between wheat productivity and biological activity and soil nitrogen content

Oleh Stasiv
Doctor of Agricultural Sciences, Director
Institute of Agriculture of the Carpathian Region of the National Academy of Sciences
81115, 5 Hrushevskyi Str., Obroshyne Village, Ukraine
https://orcid.org/0000-0003-3737-739X

Anhelina Dubytska
PhD in Agricultural Sciences, Leading Researcher
Institute of Agriculture of the Carpathian Region of the National Academy of Sciences
81115, 5 Hrushevskyi Str., Obroshyne Village, Ukraine
https://orcid.org/0000-0002-5685-0237

Oksana Kachmar*
PhD in Agricultural Sciences, Head of the Division
Institute of Agriculture of the Carpathian Region of the National Academy of Sciences
81115, 5 Hrushevskyi Str., Obroshyne Village, Ukraine
https://orcid.org/0000-0002-0382-6030

Oleksandr Dubytskyi
PhD in Biological Sciences, Senior Researcher
Institute of Agriculture of the Carpathian Region of the National Academy of Sciences
81115, 5 Hrushevskyi Str., Obroshyne Village, Ukraine
https://orcid.org/0000-0002-8293-4119

Oksana Vavrynovych
PhD in Agricultural Sciences, Leading Researcher
Institute of Agriculture of the Carpathian Region of the National Academy of Sciences
81115, 5 Hrushevskyi Str., Obroshyne Village, Ukraine
https://orcid.org/0000-0003-3466-1432

Article's History:
Received: 27.02.2025
Revised: 17.07.2025
Accepted: 27.08.2025

Abstract. Coverage of the components of causal dependencies of crop productivity mediated by N-, C-chemical, biological indicators of soil (sizes of pools of relevant available forms, enzymatic activities) in the context of theoretical aspects of resource turnover in it is potentially significant for the design of systems of evaluation criteria for soil health using machine learning (ML) models. The purpose of this study was to find statistically valid and meaningful coordination and causal relationships of grain weight per sown area with available forms of N, C (inorganic and easily hydrolysed N – NIN, NEH; C of labile humus – CLH), in situ proteolytic PA, in vitro urease UA activity in association with the concepts of the turnover of these organogens in the soil under winter wheat under prototypical environmentally friendly fertiliser systems. The

Suggested Citation:
Stasiv, O., Dubytska, A., Kachmar, O., Dubytskyi, O., & Vavrynovych, O. (2025). Causal relationships between wheat productivity and biological activity and soil nitrogen content. *Scientific Horizons*, 28(8), 30-50. doi: 10.48077/scihor8.2025.30.

*Corresponding author

following methods were employed: field, in situ, laboratory-analytical chemical and biochemical, morphometric, statistical, ML-modelling. With a high probability, the increase in GM was caused by an additive increase in NIN, NEH, and a simultaneous decrease in in situ PA, in a way that mediated the effect of in situ PA on GM by the listed forms of N. Statistically less valid was the mediation of the effect of the ratio of NEH to NIN on GM through in situ PA with possibly mutually complementary predispositions of N forms, which, however, provided the basis for substantiating the simultaneous mutually opposite determination of GM by these N forms and in situ PA. It is plausible that CLH and in vitro UA were not involved in the development of GM under these experimental conditions, but it is possible that the latter of these variables may have a more significant effect on the variation of the former than vice versa. Studies in this vein will develop a discourse of theoretical predictions regarding the improvement of the structure of soil quality indices towards revealing aspects of the laws of promoting the balance between the characteristics of agro-ecosystem services, considering the chemical and biochemical properties of N and C turnover in the soil, and crop bioproductivity necessary for the development of sustainable agriculture

Keywords: available forms of nitrogen in soil; labile humus carbon; proteolytic and urease activity of soil; winter wheat grain weight; exploratory factor analysis; systems of simultaneous equations; causal determinants

INTRODUCTION

Nitrogen (N) is a key element for primary production (PP) and soil organic matter (SOM) stabilisation in agroecosystems. Microbial activity (MA) of the soil, especially the enzymatic activity (EA) of N-mineralising enzymes (NMEs), is essential for the release of bioavailable N (BAN) from soil organic matter (OM) (Daunoras *et al.*, 2024; Wang *et al.*, 2024). According to N. Fierer *et al.* (2021), EA is a significant indicator of soil quality and health, characterising its ecological functions and ecosystem services to support bioproductivity. As noted by J. Lehmann *et al.* (2020), the analysis of NME variations is significant for assessing soil health and understanding crop production. However, a series of obstacles, such as interactions between soil components, asynchrony of N (Zhu *et al.*, 2025), etc., complicate the interpretation of changes in EA, N-forms, which also complicates the identification of the relative significance and causality of EA(NME) – BAN – bioproductivity.

Understanding the complex causal relationships between agro-technological interventions, soil characteristics, and crop productivity is key to the development of sustainable agriculture. The literature has identified two principal ways in which such links can be established. The first (*i*) involves the direct or indirect impact of agricultural technologies on soil EA, which leads to changes in SOM and nutrient release, which ultimately affect yields (Hassan *et al.*, 2022). The second (*ii*) focuses on the modification of substrates (S)/intermediates/products by agricultural technologies, which causes changes in EA, which subsequently affect nutrient accumulation, with a possible positive feedback effect and impact on yield (Tian *et al.*, 2020). The following literature review details studies that confirmed and extended these general models by considering the impact of individual factors and processes on EA, N-forms, and the interdependencies between them.

In this context, it is essential to note that S availability and nutrient limitation (LIM) are crucial factors in the regulation of NME and C- and P-metabolism enzymes. The principles of soil ecology and biochemistry expand the understanding of the interdependencies between EA and S, nutrients-LIM. Specifically, X. Tan *et al.* (2021) argued that eco-EA (EEA) of C, N, P cycles, and EE-stoichiometry (EES) depend on the state, nutrient-stoichiometry, relative LIM/availability of microbial resources, soil microbial properties, and environmental conditions. According to the theory of resource allocation, microorganisms can release exoenzymes (increase in EEA) in response to C-, N-, P-LIM to produce LIM nutrients (negative nutrient-efficiency relationships – EEA). However, there may also be an increase in EEA with the addition of S or no clear relationship. The significance of regulating NME and other enzymes is also conditioned by the fact that the uptake of bioavailable N (BAN) by crops is mainly from the soil, particularly from the turnover of SOM and soil organic nitrogen (SON) (Yan *et al.*, 2020). Therefore, the values of the balance sizes of the N_{IN}, N_{EH}, C_{LH} pools, potentially affecting GM, can be predetermined (regulated) by EA (NME). At the same time, these pools may well be involved in the regulation of isPA, ivUA (feedback loop).

Admittedly, earlier studies by Q. Ma *et al.* (2020), B. Adamczyk (2021), revealed that soil PA is a crucial LIM factor in mineralisation and N availability, although L. Greenfield *et al.* (2020) suggested that the rate of protein supply may have a greater influence on differences in N turnover. PA regulation is a complex process that is influenced by a variety of abiotic and biotic factors, such as NPP, NO_3^- concentration, C:N, and soil height and depth. T. Jesmin *et al.* (2021) noted a NH_4^+/NO_3^--repressive effect on PA, but suggested a stimulating effect of organic C-, N-compounds of plant residues on this NME. Since the SON dynamics, which is determined by SOM and MBC via MA (EA), is vital for the accumulation of mineralised S, it is natural that MBC and initial SOM directly and indirectly through PA and glutaminase affect the SON dynamics.

The mutual regulation between nutrient pools and UA is less understood. Overall, nutrient availability (NH_4^+, SOM, SON), C:N are crucial regulators of UA, where NH_4^+-sensitivity may vary between distinct groups of bacteria (Chettri *et al*., 2021). K. Ampong *et al*. (2022) also showed that the effect of humic acids on soil UA can be different and depends on their type, origin, and can be modulated by MB, C:N, nitrification/denitrification. UA can be induced by the presence of N and OM but inhibited by high concentrations of NH_4^+ as the only N source. Further mineralisation processes are also subject to complex regulation involving S/intermediates/reaction products. The key factors controlling the mineralisation rate are the quantity and quality of detritus and natural fertilisers. Furthermore, C:N in SOM plays a significant role in determining the mineralisation/immobilisation balance (Dalias & Christou, 2022). However, Y. Cao *et al*. (2021) argued that the key role in regulating the response of microbial N immobilisation to OC addition belongs to the characteristics of its chemical quality, which is determined by the ratios of labile, intermediate, and recalcitrant pools.

Nitrification, as one of the final stages of complete N-mineralisation, is also characterised by complex regulation. NH_4^+ is S for nitrifying bacteria and therefore accelerates nitrification. On the other hand, high C:N can lead to competition for N among microorganisms, decelerating nitrification. The NH_4^+ requirement of plants can also affect the rate of nitrification, as plants are the primary consumers of mineral N. O_2 is essential for aerobic nitrifying bacteria and therefore stimulates nitrification while inhibiting anaerobic denitrification. Analysing the relationships between denitrification and readily available SOC (energy source), O_2 and NO_3^-/mineral N in soil, B. Pan *et al*. (2022) found that the latter contributed to increased N_2 emissions. The ratio of $N_2O/(N_2O + N_2)$ increased with increasing soil O_2 but decreased with SOC, C:N, and soil pH. Therewith, G.P. Robertson and P.M. Groffman (2024) argued that with sufficient SOC and O_2 deficiency, NO_3^- concentration can become a LIM factor in denitrification. Global studies (structure equation modelling, SEM) conducted by Z. Li *et al*. (2020) showed that total nitrogen (TN) and microbial nitrogen (MBN) are positively correlated with nitrification, which is also related to SOM content. An increase in C-LIM (especially microbial) and a decrease in N-LIM (leading to an increase in SON) can stimulate both N-mineralisation and nitrification.

Thus, there is a complex network of interactions/interregulations and causality between biological and chemical factors that control N cycling in soil, including nutrient availability, EA, OM. Mineralisation, nitrification/denitrification are regulated by various biotic and abiotic factors, including C:N, O_2, MB. Understanding these relationships is key to assessing the influence of agroecosystems on soil productivity and stability. The purpose of this study was to determine the stochastic (inter)coordination and causal relationships between the soil traits N_{IN}, N_{EH}, C_{LH}, isPA, ivUA and GM of winter wheat under ESFS conditions.

MATERIALS AND METHODS

The study described in this publication was conducted in 2021-2022 on grey forest surface-gleyed sandy-loam soil under winter wheat (*Triticum aestivum* L.) of the Benefis variety (stationary experiment to investigate the scientific basis of productivity management of short-term crop rotations in the Carpathian region; Institute of Agriculture of the Carpathian Region of the National Academy of Agrarian Sciences of Ukraine). The experimental microplots (area = 1 m^2; replication = 3) were arranged systematically; the distance between them was 0.5 m. Before setting up the experiment, the key physicochemical and agrochemical parameters of the soil were determined (depth 0–30 cm): pH_{KCl} = 4.78-4.92, hydrolytic acidity (according to Kappen) 23.83-24.61 mg-eq kg^{-1} of the soil, the content of the easily hydrolysed nitrogen (N_{EH}; according to Kornfield) 86.22-91.32 mg kg^{-1} of the soil, available phosphorus, available potassium (according to Kirsanov; extraction with 0.2 M HCl) – 105.11-113.04 mg kg^{-1} of the soil and 84.03-90.17 mg kg^{-1} of the soil, respectively, the carbon (C) content of total humus (according to Tyurin) – 1.91-1.96%. All experimental variants (ecologically safe technologies) were divided into 2 groups (each with its own control), depending on the applied precursor: *i*) variants of group A (A-1 – A-7); the precursor was pea (*Pisum sativum* L.); *ii*) variants of group B (B-1 – B-7); the precursor was fodder bean (*Vicia faba* L. var. major Harz). The layout of the typical agrotechnical and ecologically safe factors in the listed variants 1-7 of groups A, B was the same, and was as follows: A-1) control (no mineral or organic fertilisers or ecologically safe factors were used; the precursor was pea); A-2) FPS (field pea straw); A-3) FPS + RMF (recommended rate of mineral fertilisers – $N_{90}P_{60}K_{60}$; A-4) FPS + RMF + BS (biostimulant "Millerplex"); A-5) FPS + RMF + BS + HF (humic fertiliser "BlackJak"); A-6) FPS + RMF + CF (chelated fertiliser of the "Rozalik" range); A-7) FPS + HMF (high rate of mineral fertilisers – $N_{150}P_{120}K_{120}$) + HF; B-1) control (mineral or organic fertilisers or the ecologically safe factors were absent; precursor – fodder beans); B-2) FBS (fodder beans straw); B-3) FBS + RMF (recommended rate of mineral fertilisers – $N_{90}P_{60}K_{60)}$; B-4) FBS + RMF + BS (biostimulant "Millerplex"); B-5) FBS + RMF + BS + HF (humic fertiliser "BlackJak"); B-6) FBS + RMF + CF (chelated fertiliser of the "Rozalik" range); B-7) FBS + HMF (high rate of mineral fertilisers – $N_{150}P_{120}K_{120}$) + HF. A brief description, methods, and time of application of the commercial preparations used in the study – components of the ecologically safe factors (bioeffectors), are presented below.

The biostimulant (BS) "Millerplex". Treatment of plants twice during the spring-summer vegetation in the phase of spring tillering, tubing at 0.5 L ha^{-1}. Contains natural cytokinins from 3 plant sources, which have enhancing properties for the hormonal regulation of growth of winter cereals at the cellular level, particularly under stressful conditions. Composition of the preparation: N amide – 3.0%, P_2O_5 – 3%, K_2O – 30%, algae extract *(Ascophyllum nodosum*); pH – 6.0-6.5; additionally, amino acids, specific carbohydrates (improve the immune system of plants), trace elements (Cu, Fe, Mn, Mg, Zn, B, Co) in chelated form. Built-in adjuvant system for maximum penetration and translocation of biological components into plants. The humic fertiliser (HF) "BlackJak". Spraying on the surface of the root zone twice during the spring-summer growing season in the phases of spring tillering and tubing at 1.0 L ha^{-1}. Has a prominent biostimulant effectiveness; apart from humic and fulvic acids, it contains ulmic acids and humin (the most active in plants). Composition: humic acids – 19-21%, fulvic acids – 3-5%, total organic matter – 27-30%; pH – 3.5-5.0. The product is effective both for plants and is most useful in the soil. The chelated fertiliser (CF) of the "Rozalik" range (Zn, P, N, S). Treatment of plants twice during the spring-summer growing season in the phases of spring tillering, tubing at 2.0 L ha^{-1}. Composition: N amide – 3%, P_2O_5 – 19%, SO_3 – 5.3%, Zn in the Zn-EDTA form – 5.9%.

The dynamics of synoptic conditions during the autumn of 2020 and the growing seasons of 2021-2022 was characterised by a combination of the following properties: *i*) the winter and spring-summer intervals of 2020-2021 and 2021-2022 differed in terms of precipitation (PI); in the winter-summer period of 2021, PI was 473.6 mm, while in 2022 it was 318.3 mm (average long-term PI values were 458.0 mm); more than half of these precipitations occurred in the winter-spring interval; *ii*) in 2022 there was a slight lack of moisture during March, May-June, which, however, did not affect biological processes in the soil, plant development, and their final productivity; *iii*) during the entire time interval of the study, the temperature regime of the soil and atmospheric air was predominantly favourable for biological processes in the substrate, for the growth and development of winter wheat; overall, the average monthly temperatures were comparable during the spring and summer periods of 2021 and 2022; the exception was July, when the air temperature was 4.7°C greater than the long-term average. Thus, meteorological conditions during the autumn-winter and spring-summer growing seasons of the above years were generally satisfactory for biological processes in the soil and the development of winter wheat bioproductivity.

After the onset of the ontogenetic stages of spring tillering and wax ripeness (~75% of plants in one of these stages), soil samples were collected (depth 0-30 cm; row spacing – bulk soil) from experimental variants A-1–A-7, B-1–B-7, and prepared for *in vitro* testing of the soil traits described in this study (DSTU ISO 11464-2001, 2003). The content of $N\text{-}NO_3^-$ was determined potentiometrically according to DSTU 4725:2007 (2008), $N\text{-}NH_4^+$ – using Nessler's reagent (DSTU 4729:2007, 2008); the total content of the 2 listed forms of nitrogen in the soil was denoted as inorganic nitrogen $N_{IN} = N\text{-}NO_3^- + N\text{-}NH_4^+$. The content of easily hydrolysed nitrogen (N_{EH}, according to Kornfield) was measured according to DSTU 7863:2015 (2016), the content of carbon of labile humus (C_{LH}, soluble in 0.2 M NaOH) was measured according to the Tyurin method – DSTU 4732:2007 (2008). N_{IN}, N_EH, C_{LH} were calculated at mg kg^{-1} of air-dried soil. To prepare the Nessler's reagent, the study used the corresponding procedure presented in S. Abdelwahed *et al.* (2021), which the authors of the present study modified as follows: 45.50 g of HgJ_2 "CHDA" (Chemlaborreactor LLC) was mixed with 35.00 g of KJ "CHDA" (Sphere Seven LLC), 50.00 ml of double-distilled water was added, and gradually stirred in 112.00 g of cold KOH "CHDA" (Sphere Seven LLC) + double-distilled water until the components were completely dissolved; stirring and cooling, the volume was gradually brought to 800.00 ml, and then to 1.00 L; left in the dark for ~72 h. The resulting reagent was filtered (ashless filter "Blue Ribbon"); therewith, the neck of the funnel contained glass wool, previously washed with dilute solutions of HCl, distilled water, and double-distilled water.

To determine the *in vitro* urease activity (urease, urea aminohydrolase, EC 3.5.1.5) in soil (ivUA) (0-30 cm; row spacing – bulk soil), the study employed typical approaches for testing this activity in soils, plant seeds, microorganisms, specifically, soil treatment with toluene (Cheng *et al.*, 2020; Tahir *et al.*, 2023), a reaction mixture based on 0.2-0.5 M phosphate-buffered saline (PBS) with pH ≈ 7.0, incubation temperature 30-38°C, incubation time 24 h (Zusfahair *et al.*, 2021; Magomya *et al.*, 2022), photometric measurement of NH_4^+ concentration with Nessler's reagent at $\lambda \approx 400$ nm (Nathan *et al.*, 2020). Briefly, ~5.00 g of air-dry soil, 10.00 ml of PBS (pH 6.7), 0.50 ml of toluene, 10.00 ml of 10.00% urea solution were added to an Erlenmeyer flask, sealed with cork stoppers, incubated at 30°C for 24 h; after incubation, 15.00 ml of 1 M KCl solution were added (reaction stop + NH_4^+ displacement); the contents of the tube were centrifuged at 3×10^3 rpm; an aliquot of the supernatant was significantly diluted with double-distilled water in analytical tubes, and a solution of K, Na-tartrate was added (the last 2 steps were performed in such a way as to leave room only for the Nessler reagent); 5–10 min after the addition of Nessler's reagent, the extinction of the solution was measured at $\lambda = 400$ nm using an electrophotocolourimeter CFC-2. The value of ivUA was calculated in mg NH_4^+ kg^{-1} of air-dried soil hour^{-1}. The toluene, urea, K, Na-tartrate used in this method were of AR grade (Sphere Seven LLC), the phosphate components of PBS, KCl were of chemically pure grade (Sphere Seven LLC); all solutions were prepared with double-distilled water.

To determine the *in situ* proteolytic activity (isPA) in the field using the method of Mishustin, Nikitin, and Vostrov (described by Khaziev) (Dubytska *et al.*, 2022), a gelatine-containing coating of medical X-ray film was used as a substrate. The film was placed vertically in the soil (up to 30 cm deep from the bottom edge; row spacing) and left for 4–8 days, pressed down from the sides (so that the middle of this time interval was approximately at the beginning of the onset of the ontogenesis phases of spring tillering and wax ripeness, i.e., ~75% of plants in one of these phases). After the film was excavated, it was placed in chilled plastic containers and transported to the laboratory. Before and after excavation, the film was weighed on a Radwag AS 220/R2 analytical balance (Poland) (±0.0001 g); before weighing the film with the already fermented gelatine-containing coating, the substrate carrier was lightly washed from the soil with a moderate stream of tap water and then air-dried for several hours. The isPA was calculated as the ratio of the film weight after and before application, expressed as %%. All the above measurements were performed in 3 analytical (or "field" for isPA) parallels, and in 3 replicates ($n = 6$). The obtained individual data for each indicator in the ontogenetic phases of spring tillering and wax ripeness were averaged among themselves for the spring and summer vegetation in 2021 or 2022. Thus, sets of averaged individual data ($n = 6$) were obtained for the corresponding periods of plant development for years 1 and 2.

In grassroots approaches, such as genetic studies, at least 8–10 random or representative plants per plot or replication are sufficient to record ear productivity of agricultural cereal plants, specifically, grain weight per ear, *etc.* (Zhai *et al.,* 2020). In the present study, after the full grain ripeness stage (~75% of plants), 6 ears of grain were selected from productive shoots of winter wheat plants per replication, and in 3 replications ($n = 18$) to estimate the grain weight per plant. Grain weight was measured on an analytical balance (±0.0001 g) (Radwag AS 220/R2, Poland). Grain mass per sown area (GM, kq m^{-2} of the sowing area) was calculated as the product of grain mass per plant and the number of productive shoots per m^2. The final values of each soil and plant trait for the 2 years of research were obtained by averaging the corresponding values (for $n = 6$ or $n = 18$; year 1, year 2) for the spring-summer growing season in 2021 and 2022.

The assessment of statistical validity p of pairwise differences between the numerical sizes of the analysed indicators in experimental variants A-1–A-7, B-1–B-7 was assessed using the Student's t-test calculated as described by S. Brown *et al.* (2020); the appraisal of statistical soundness p of differences between the data in the groups of experimental variants A-1–A-7, B-1–B-7, and in the combined category A-1–B-7 were performed using ANOVA; the grades listed were executed in Microsoft Office Excel 14.0.7128.5000. The indicated differences were considered significant if $p < 0.05$. The construction and analysis of the full form of the general linear model (GLM) and its not full form (without factor(s)) for the dependence of GM on the continuous predictors (covariates – random effects of variation N_{IN}, N_{EH}, C_{LH}, isPA, ivUA), categorical variable (factor – fixed effects of variation caused by experimental variants A-1–A-7, B-1–B-7) were performed on non-standardised individual (the continuous predictors) and neighbourhood averages (the categorical variable) in the experimental variation series of each experimental variant using Statistica Version 10 (StatSoft Inc). The corresponding data were presented only for the not full form of GLM. The Pearson product-moment correlation coefficient (PMC; based on the arithmetic means of each experimental variant), the corresponding p values, exploratory factor analysis (EFA) with dependent variable (GM) for 6 and 2 factors (based on the standardised means for each experimental variant; extraction – principal component analysis (PCA); rotation – varimax normalised) were calculated in the same software package. For the above methods of factor extraction and rotation, the study employed the options built into Statistica Version 10 (StatSoft Inc.) (specifically, Statistics → Multivariate exploratory techniques → Factor analysis → Variable selection → … → Advanced → Extracting method: Principal component analysis → … → Factor rotation: Varimax normalised). To obtain a variation series of 6 individual GM data per experimental variant, in the original variation series of 18 individual GM data per variant, each 3 neighbouring individual data were averaged.

The results of the EFA (2 factors) served as the basis for a preliminary prediction of potential causal relationships between the variables under study. For a more detailed assessment of the hypothesised causal relationships between predictors and response functions, simultaneous equations models (Eqs.), designated as SIEMs (from non-standardised averages for each trial) in the GNU Regression, Econometrics and Time-series Library package (GRETL 2021b; a free Unix-like operating system developed by the GNU Project); only recursive SIEMs among them were selected for the categories of resulting SIEMs. Since the 3-stage least squares (3SLS) approach to SIEMs was found to be heteroscedastic in some cases (the Breusch-Pagan test), the authors of the present study employed the weighted least squares (WLS) method (i.e., essentially, the weighted two-stage least squares W2SLS method) with iterative estimation. Additionally, in the case of 3SLS models, significant over-identification (the Hansen-Sargan test) was demonstrated several times. For the final SIEMs, the 2-stage least squares (2SLS) method provided mostly almost the same estimates as WLS, but the latter approach was still more reliable. Only the estimates built into GRETL 2021b were used as typical estimates of the statistical effectiveness of SIEMs. The values of Fisher's Criterion (FC) and their statistical validity for each Eq. in SIEMs were estimated in Microsoft

Office Excel 14.0.7128.5000 according to D.M. Levine *et al.* (2021). During the theoretical analysis, it became necessary to combine 2 Eqs. with semantic and mathematical mutual inverse causality within one system (e.g., C_{LH} → ivUA, ivUA → C_{LH}). The analysis of such SIEM using seemingly unrelated regressions (SUR), 2SLS, 3SLS, WLS approaches did not reveal any successful variance models. In this regard, two types of independent Eqs. were found (C_{LH} → ivUA; ivUA → C_{LH}) using the Ordinary Least Squares (OLS) method (OLM) in GRETL 2021b. In addition, combining the last of the listed types of Eqs. into a single model within the assumed "subordination" using the 4 methods listed above did not lead to the formation of valid SIEMs. Therefore, the independent Eqs. derived by OLM were fully analysed (including FC) in GRETL 2021b and left for further interpretation. The study followed the standards of the Convention on Biological Diversity (1992) and the Convention on Trade in Endangered Species of Wild Fauna and Flora (1979).

RESULTS

The study found that under the conditions of the environmentally safe technologies in the soil of variants of groups A (A-2–A-7) and B (B-2–B-7), there was a significant (p < 0.001-0.05) increase in the content of N_{IN} by 8.93-64.36%, 5.37-86.19%, relative to the control variants A-1, B-1, respectively (Fig. 1, a). Analogously, statistically significant (p < 0.001) increases in N_{EH} content in the variants of groups A-2–A-7 and B-3–B-7, compared to the indicated controls, were 5.86-34.59%, 21.26-29.50%, respectively. In the case of B-2, the last of the listed indicators did not undergo statistically significant changes (p > 0.01), compared to B-1. The values of both of the listed agrochemical features of soil N-state were statistically significantly (p < 0.001) greater by 23.68-76.69% (for N_{IN}), 21.06-29.49% (for N_{EH}) under the conditions of the ecologically safe technologies A-3–A-7, B-3–B-7 *vs.* A-2, B-2, respectively (Fig. 1, a). Furthermore, in the variants A-2–A-7, B-2–B-7, there was an increase in the level of C_{LH} by 5.73–39.17%, 7.93-41.38% (p < 0.001–0.01), relative to A-1, B-1, respectively (Fig. 1, b); in addition, the ecologically safe technologies A-3–A-7, B-3–B-7 caused an increase in the size of this indicator by 3.92-31.63%, 4.15-30.99% (p < 0.001-0.05) compared to A-2, B-2, respectively.

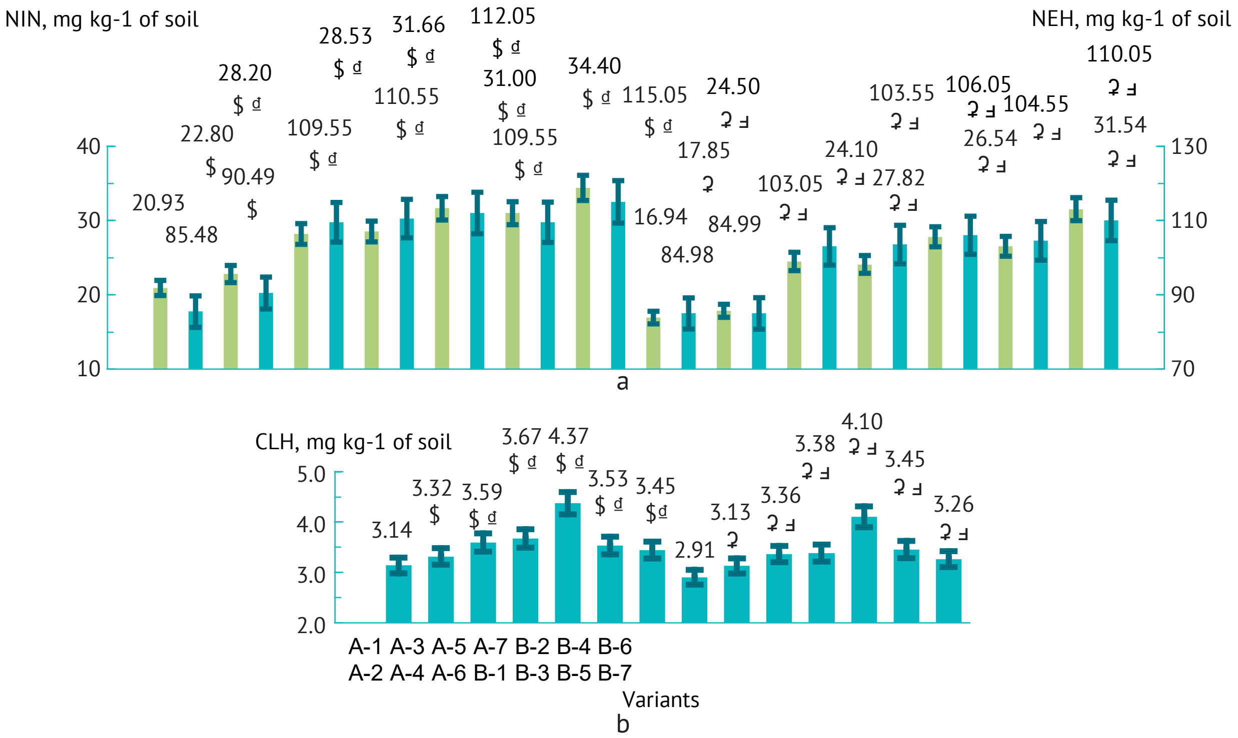


***Figure 1**. Variability of inorganic nitrogen (N_{IN}), easily hydrolysable nitrogen (N_{EH}) (a) and C of labile humus (C_{LH}) (b) in the soil under winter wheat under the research variants*

***Note:** A-1, B-1 – the controls of variant groups A, B with pea and fodder beans as precursors, respectively; A-2–A-7 or B-2–B-7 – the ecologically safe technologies of variant groups A or B, respectively; column height in the diagram = arithmetic mean (mean, M; n = 6; data averaged by ontogenetic stages spring tillering, wax ripeness, and for 2021–2022), error bars = ± standard error (SE, m); numbers next to the levels of M ± m – numerical values of the corresponding means; $ or ʡ – the statistical validity of differences between the study variants A-2–A-7 and A-1 or B-2–B-7 and B-1 – p < 0.001–0.05; ₫, ɟ – the statistical reliability of differences between the variants A-3–A-7 and A-2 or B-3–B-7 and B-2 – p < 0.001–0.05; the statistical soundness of differences between the data in ANOVA for A-1–A-7, B-1–B-7 – p < 0.001–0.003, A-1–B-7 – p < 0.001*

***Source:** developed by the authors of this study*

All the ecologically safe technologies used in variants A-2–A-7, B-2–B-7 caused a statistically significant ($p < 0.001$) increase in isPA by 1.50-9.35%, 2.05–9.40% *vs.* A-1, B-1 controls, respectively (Fig. 2, a); at the same time, the increase of the indicated soil EA under conditions A-3–A-7, B-3–B-7 by 3.40-7.85%, 2.91-7.35% (p 0.001) was found compared to A-2, B-2, respectively. Additionally, the ecologically safe factors in variants A-2–A-7, B-2–B-7 caused a statistically significant ($p < 0.001$) increase in ivUA by 93.69-404.50%, 120.19-522.12%, relative to controls A-1, B-1, respectively (Fig. 2, b); ivUA increases in A-3–A-7, B-3–B-7 when compared with A-2, B-2, were 74.81-160.47%, 78.78-182.53% ($p < 0.001$), respectively.

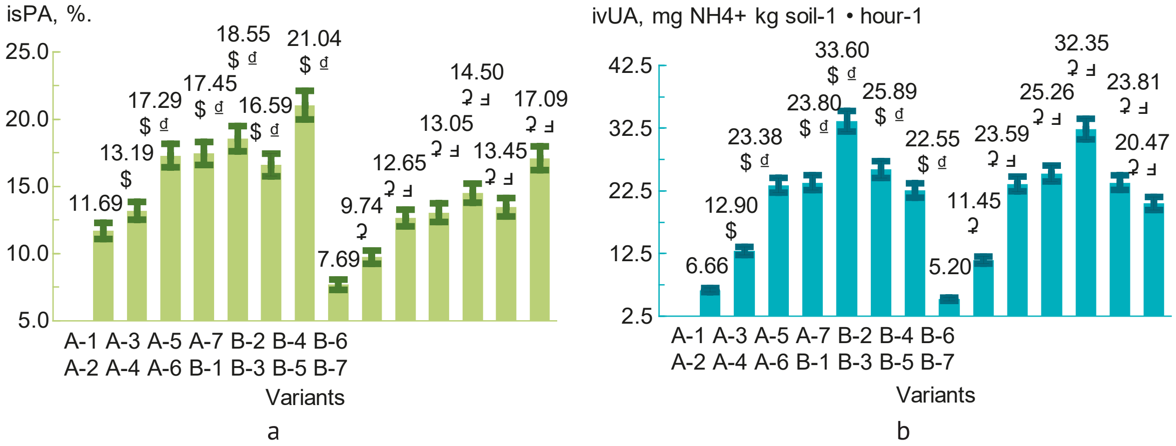


***Figure 2**. Changes in the arithmetic averages of in situ proteolytic activity (isPA) (a) and in vitro urease activity (ivUA) (b) in the soil under winter wheat depending on the research variant*

***Note:** A-1, B-1 – the controls of variant groups A, B with pea and fodder beans as precursors, respectively; A-2–A-7 or B-2–B-7 – the ecologically safe technologies of variant groups A or B, respectively; column height in the diagram = arithmetic mean (mean, M; n = 6; data averaged by ontogenetic stages of spring tillering, wax ripeness, and for 2021–2022), error bars = ± standard error (SE, m); numbers next to the levels of M ± m – numerical values of the corresponding means; $ or ʡ – the statistical reliability of differences between the variants A-2–A-7 and A-1 or B-2–B-7 and B-1 – p < 0.001; đ̲, ɟ – the statistical validity of differences between the variants A-3–A-7 and A-2 or B-3–B-7 and B-2 – p < 0.001; the statistical soundness of differences between the data in ANOVA for A-1–A-7, B-1–B-7, A-1–B-7 – p < 0.001*

***Source:** developed by the authors of this study*

Each of the ecologically safe technologies used in variants A-2–A-7, B-2–B-7 provided a statistically significant ($p < 0.001$) increase in GM within 14.29-159.11%, 17.55-163.30%, compared to controls A-1, B-1, respectively (Fig. 3); the ecologically safe factors in variants A-3–A-7, B-3–B-7 caused the levels of increment of this indicator by 49.57-126.72%, 50.23-123.98% ($p < 0.001$), relative to A-2, B-2, respectively.

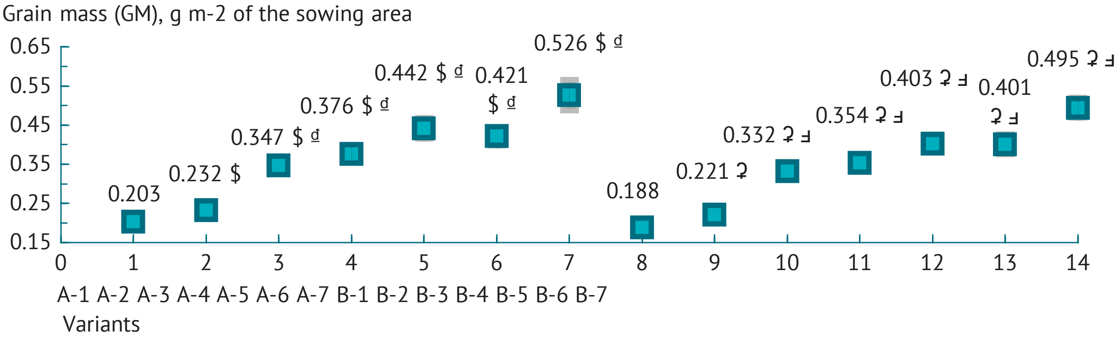


***Figure 3**. Grain mass (GM) per sowing area of winter wheat depending on the research variant*

***Note:** A-1, B-1 – the controls of variant groups A, B with pea and fodder beans as precursors, respectively; A-2–A-7 or B-2–B-7 – the environmentally safe technologies of variant groups A or B, respectively; "shaded square" symbols – the arithmetic mean (mean, M; n = 18; ontogenetic phase of wax ripeness; averaged data for 2021-2022), error bars = ± standard error (SE, m); numbers next to the "shaded square" symbols – numerical values of the corresponding means; $ or ʡ – the statistical validity of differences between the research options A-2–A-7 and A-1 or B-2–B-7 and B-1 – p < 0.001; đ̲, ɟ – the statistical reliability of differences between the variants A-3–A-7 and A-2 or B-3–B-7 and B-2 – p < 0.001; the statistical soundness of differences between the data in ANOVA for A-1–A-7, B-1–B-7, A-1–B-7 – p < 0.001*

***Source:** developed by the authors of this study*

The one-factor (paired) linear correlation analysis showed the presence of statistically valid ($p \leqslant 0.001$-0.05) medium and close interdependencies between GM and N_{IN}, N_{EH}, C_{LH}, isPA, ivUA with PMC values of 0.5500-0.9473 (Table 1). Therewith, the PMC values for the mutual subordinations between the last 5 variables listed were also significant and statistically reliable (0.6055-0.9441; $p \leqslant 0.001$-0.05). Such results most likely reflect multicollinearity within the correlation matrix under study. This substantially hinders the development of even the conceptual configuration of the causal relationships between GM and the N- and C-soil indicators presented here.

***Table 1**. Estimates of effect sizes, parameters, power, and efficiency for the not full form of GLM dependence of GM on the studied soil indices and the Pearson product-moment correlation coefficient (PMC) between these traits*

Index	GLM (n = 6 × 14)				Product-moment correlation coefficient (PMC)				
	Index								
	Partial η^2	Parameter	β	OP (α =0.05)	GM	N_{IN}	N_{EH}	C_{LH}	isPA
Intercept	0.5498$^{\text{æ}}$	582.202$^{\text{æ}}$	–	1.0000$^{\text{æ}}$	–	–	–	–	–
N_{IN}	0.1607$^{\text{æ}}$	24.273$^{\text{æ}}$	1.217$^{\text{æ}}$	0.9684$^{\text{æ}}$	0.9473$^{\text{æ}}$	-	–	–	–
N_{EH}	0.1285$^{\text{æ}}$	-4.871$^{\text{æ}}$	-0.638$^{\text{æ}}$	0.9176$^{\text{æ}}$	0.9347$^{\text{æ}}$	0.9441$^{\text{æ}}$	–	–	–
C_{LH}	0.3232$^{\text{æ}}$	-155.755$^{\text{æ}}$	-0.711$^{\text{æ}}$	1.0000$^{\text{æ}}$	0.5500$^{\text{č}}$	0.6224$^{\text{č}}$	0.6500$^{\text{č}}$	–	–
isPA	0.0015	-2.358	-0.078	0.0630	0.8607$^{\text{æ}}$	0.9608$^{\text{æ}}$	0.8965$^{\text{æ}}$	0.6055$^{\text{č}}$	–
ivUA	0.4019$^{\text{æ}}$	10.027$^{\text{æ}}$	0.743$^{\text{æ}}$	1.0000$^{\text{æ}}$	0.7733$^{\text{æ}}$	0.7553$^{\text{æ+}}$	0.8567$^{\text{æ}}$	0.8669$^{\text{æ}}$	0.6732$^{\text{þ}}$

	Separate performance indicators for the not full form of GLM
R^2	0.7564
R^2(Adj)	0.7408
FC	48.4435
p (GLM)	< 0.01

***Note:** GLM – General Linear Model (6 data per variant; 14 variants); β – standardised regression coefficient; OP – observed power; R^2, R^2 (Adj), FC, p (GLM) – the multiple coefficient of determination, adjusted multiple coefficient of determination, the Fisher coefficient, and the statistical reliability of the GLM; GM, NIN, NEH, CLH, isPA, ivUA – the grain mass, the inorganic N, the easily hydrolysed N, the carbon of labile humus, in situ proteolytic activity, the in vitro urease activity; æ, æ +, þ, č – the statistical validity of $p \leqslant 0.001$, $p = 0.002$, $p \leqslant 0.01$, $p \leqslant 0.05$, respectively*

***Source:** developed by the authors of this study*

The authors of this study suggested that for a better understanding of the systemic principles of the probable way GM is subordinated to N_{IN}, N_{EH}, C_{LH}, isPA, ivUA, it may be potentially expedient to generate the full form of the GLM (data not presented; the reasons for this are explained below) and/or the not full form of the GLM (without categorical variable); the results of the latter analysis are presented in Table 1. On the other hand, the relevance of this approach is confirmed by the properties of the structure of the used experimental data, namely: *i)* the above-mentioned multicollinearity within the correlation matrix of the data; *ii)* the deviation from the normal distribution of the N_{EH} parameter (*p* for H_1 "there is a deviation from the normal distribution (ND)" in the case of the Doornik-Hansen, Lilliefors, and Shapiro-Wilk W-statistic tests was 0.0130-0.0222; in this situation, only the Jarque-Bera test revealed $p = 0.4017$). Notably, logarithmisation of the data exacerbated the situation with the ND for $\ln(N_{EH})$ and caused deviations from the ND for ln(ivUA) (approximately the same as in the case of the aforementioned mean variable).

In the resulting full form of the GLM ($R^2 = 0.8351$, $R^2_{(Adj)} = 0.7894$, $p < 0.01$; data not presented), only the values of the partial η^2 for the intercept (= constant = b_0) and the categorical variable (effects of variation due to the experimental variants) were statistically valid ($p < 0.001$-0.05); the rest of the values of the partial η^2 for the independent variables were statistically insignificant ($p = 0.7162$-0.9795). In this form of GLM, the values of the parameters (regression coefficients b_0, b_i) for the intercept = b_0 and the categorical variable for options A-1, A-2, A-7, B-1, B-2 were statistically valid ($p < 0.001$-0.05). Therewith, the statistical significance of the variability of all the variables under study by variants was proved earlier using ANOVA (outside the full form of the GLM) and, on the other hand, the full form of the GLM was unable to characterise the subordination of GM to the predictors presented herein. The analysis of the not full form GLM showed significantly higher values of the partial η^2, parameter = b_0 for the intercept, compared to the partial η^2, b_i for the other regressors, and at the same time, statistical unreliability of the partial η^2, b_4 for isPA (Table 1). Clearly, this means that GM is more dependent on GM properties than on the regressors, and that GM is not caused by the variation of isPA. The synergistic effect of N_{IN}, ivUA, and the antagonistic effect of N_{EH}, C_{LH} on GM development (by the signs of the parameter) are also evident

(Table 1). Therewith, certain inconsistencies between the not full form GLM and the correlation matrix of the data, as well as the lack of opportunities to predict the mediating effects of predictors, limit the potential of using this model to establish causality within the analysed system of variables.

The next step towards predicting causality was to apply EFA (with GM as the dependent variable) to find indicators that had the following set of properties: *i)* they shared variance with each other and simultaneously with GM; *ii)* their variance was separated from the unique variance and *iii)* from the error variance (Schreiber, 2021; Watkins, 2021). Furthermore, M.S. Sulaiman *et al.* (2021) demonstrated that regression of variables on the principal components (PCs) or the rotated PCs (essentially, on the factors in the FA) is characterised by a lower level of muticollinearity compared to regression of the original data; moreover, R^2 is greater in the second case. Thus, EFA can reduce multicollinearity between variables. For factor extraction in this study, PCA was employed to reduce the number of items to a smaller number of representative uncorrelated components and to account for a larger proportion of the variance. Earlier, N. Shrestha (2021) confirmed that PCA solutions can lead to the emergence of items with cross-loadings; the variance of such an item can be explained by several factors. However, the authors of the current study considered it possible to ignore this drawback to achieve the "essentially" desired purpose of EFA here. Based on the theoretical hypothesis that a simple latent structure should be at least partially based on uncorrelated measurements, the factor loadings matrix was orthogonally rotated using the varimax normalised method. An item was considered a statistically relevant factor identifier if it had a loading of ⩾ 0.7 (Watkins, 2021).

The EFA results for the 6 factors (= the number of all variables) are presented in Table 2. The eigenvalues (EV) for the factors FA-1–FA-6 were 4.9895, 0.6947, 0.2206, 0.0673, 0.0148, 0.0130, and the corresponding values of the total variance (%) for the eigenvalues were 83.16, 11.58, 3.68, 1.12, 0.25, 0.22. For almost all variables within FA-1–FA-4, cross-loadings were found in the range of debatable values ~0.3-~0.6 (Watkins, 2021), and no such loadings were found with values closer to significant (e.g., 0.60-0.69). As mentioned above, this may be conditioned by the PCA extraction of the factors, but it is also possible that it may be caused by the properties of the original data, specifically, multicollinearity between the analysed features (Kyriazos & Poga, 2023). However, statistically valid loadings occur only in the case of FA-1 and FA-2. Additionally, the EVs and total variance values for FA-1–FA-6 indicated the presence of a "breakpoint" (Schreiber, 2021) immediately after FA-1 or, at least, after FA-2, while the software used suggested leaving only FA-1 and FA-2.

***Table 2**. Loadings of the first six factors (the standardised variables; extraction: PCA; rotation: varimax normalised)*

Variable	Factor loadings					
	FA-1	FA-2	FA-3	FA-4	FA-5	FA-6
GM	0.8062*	0.2590	0.4330	0.3088	0.0018	0.0106
N_{IN}	0.8959*	0.3398	0.2526	0.0861	0.0152	0.1026
N_{EH}	0.7749*	0.3739	0.4967	-0.0054	-0.1140	-0.0043
C_{LH}	0.2970	0.9453*	0.1288	0.0326	-0.0229	0.0012
isPA	0.9301*	0.3371	0.1034	-0.0664	0.0204	-0.0756
ivUA	0.4059	0.7116*	0.5688	0.0494	0.0536	0.0114
Expl.Var.	3.1710	1.8359	0.8489	0.1107	0.0170	0.0165
Prp.Totl.	0.5285	0.3060	0.1415	0.0184	0.0028	0.0027

***Note:** GM – grain mass (dependent variable); N_{IN}, N_{EH}, C_{LH}, isPA, ivUA – inorganic N, easily hydrolysed N, labile humus carbon, in situ proteolytic activity, in vitro urease activity (independent variables); FA-1–FA-6 – extracted factors; * – loadings > 0.700000*

***Source:** developed by the authors of this study*

Elimination of FA-3 to FA-6 resulted in an increase in the amount of explained variance and the share of total variation for FA-1 and FA-2 (Table 3). The insignificant cross-loadings for FA-1 for the variables C_{LH}, ivUA and for FA-2 for the variables GM and N_{IN}, N_{EH}, isPA were within 0.2833–0.5148. The authors of this study considered it expedient not to involve the last of these elements in the interpretation to preserve the possibility of forming and explaining simple, meaningful and mathematical constructs for the desired causal relationships between the variables. Even though the ratio between the EVs for FA-1 and FA-2 (~7.182 > 3) (Schreiber, 2021) was found to be low (potentially, it may represent a relatively small part of the total variance, which, while not directly attributable to the model, can affect the observed estimates of such a construct in the experimental conditions and in reality). It is likely that in this case, the subordinations between the statistically reliable FA-2 items (C_{LH}, ivUA) are at least partially independent of the key causal relationships between FA-1 items (GM, N_{IN}, N_{EH}, isPA).

***Table 3**. Loadings of the first two factors (extraction: PCA; rotation: varimax normalised), obtained based on the standardised variables*

Variable	Factor loadings	
	FA-1	**FA-2**
GM	0.9065*	0.3370
N_{IN}	0.9205*	0.3666
N_{EH}	0.8604*	0.4661
C_{LH}	0.2833	0.9364*
isPA	0.8975*	0.3243
ivUA	0.5148	0.8228*
Expl.Var.	3.5600	2.1243
Prp.Totl.	0.5933	0.3540

***Note:** GM (dependent variable) – grain mass; N_{IN}, N_{EH}, C_{LH}, isPA, ivUA (independent variables) – inorganic N, easily hydrolysed N, labile humus carbon, in situ proteolytic activity, in vitro urease activity; FA-1, FA-2 – extracted factors; marked (*) loadings > 0.700000*

***Source:** developed by the authors of this study*

Considering that the general organisation of the causal relationships between the variations of N-, C-soil indicators and GM studied herein under conditions of "central" mediation involving N_{IN} (and/or N_{EH}) or isPA (and/or ivUA), proposed in the INTRODUCTION section, is partly based on experimental data, partly on theoretical findings, and itself constitutes a certain theoretical generalisation, the following ways to elucidate the detailed structure of such subordination could be possible, namely: *(i)* exploratory structural equation modelling (ESEM) and its variants; *(ii)* confirmatory factor analysis structural equation modelling (CFA/SEM) and its variants; and *(iii)* path analysis models. However, the inclusion of latent constructs, complex types of variances, and combinations of mediator, moderator conditions between them, etc., in ESEM, CFA/SEM is more suitable for creating models on sufficiently large samples with a prominent level of generalisation of the final mathematical construction. Therewith, a series of simplifications in path analysis approaches, again, contribute too much to the "generalisation" of the resulting model. In the case of the more "local" patterns studied herein and the use of rarely used integrated environmentally safe technologies, it would be advisable to first characterise the causal relationships between variables with a somewhat greater level of specificity (as it were, narrowing down generalisations).

In this regard, the resulting factor structure (Table 3) was interpreted in the following 2 stages. Firstly, it is reasonable to assume that between the variables that significantly load (> 0.7) the factor 1, there may, overall, be causal subordination, where (*i*) N_{IN} (or N_{EH}) are predetermined and mediate the effects of each other and/or isPA on GM under the conditions of simultaneous action of N_{IN}, N_{EH}, isPA on GM, or (*ii*) not only N_{IN} (or N_{EH}) (mediating each other's effects), but also isPA (mediating the effects of both N_{IN} and/or N_{EH}) are predetermined, provided that N_{IN}, N_{EH}, isPA act simultaneously on GM. Secondly, variables that reliably load (> 0.7) the factor 2 can be involved in semantic and mathematical reverse causality: variation in C_{LH} causes the development of ivUA, and vice versa, variability in ivUA leads to the formation of C_{LH}. Clearly, the above concept of causality between the above N- and C-soil condition traits and the trait of final biological productivity of winter wheat differs in a series of general points and nuances from the one presented in the INTRODUCTION section. A brief presentation of the finally proposed systems of causal subordination is as follows. Firstly: (*i*) N_{IN}, N_{EH}, isPA → *GM*; isPA, N_{EH} → N_{IN} (or isPA, N_{IN} → N_{EH}); isPA → N_{EH} (or isPA → N_{IN}), respectively; (*ii*) N_{IN}, N_{EH}, isPA → GM; N_{IN}, N_{EH} → isPA; N_{EH} → N_{IN} (or N_{IN} → N_{EH}), respectively. Secondly: C_{LH} → ivUA (and/or ivUA → C_{LH}).

All the above groups of causality were estimated by 3SLS, 2SLS, WLS, and, if necessary, SUR, OLS (the latter for the group of independent Eqs. (IEs) for the 2nd stage) using non-standardised data (specifically, to approximate concrete experimental conditions) to generate mathematically relevant, interpretable SIEMs or IEs. All SIEMs and IEs that characterise the causal and only mathematical relationships, respectively, between the studied soil and winter wheat traits listed in the previous paragraph are presented below. The SIEM-1 contains Eqs. (1), (2), (3):

$$\ln((0.5260 \times 50)/GM) = 15.0370^{a} - 1.4308^{c} \times \times \ln N_{IN} - 1.6750^{c} \times \ln N_{EH} + 0.6445^{d} \times \ln isPA, \quad (1)$$

$$\ln((34.4050/N_{IN}) \times 100) = 9.9197^{a} - 0.4669^{a} \times \times \ln isPA - 0.8193^{b} \times \ln N_{EH}, \quad (2)$$

$$\ln((115.0500/N_{EH}) \times 100) = 5.7005^{a} - 0.3667^{a} \times \ln isPA, \quad (3)$$

where $^{a}, ^{b}, ^{c}, ^{d}$ – $p \leqslant 0.001, 0.01, 0.05, 0.1$.

Here and in the following Eqs., the symbol "ln" means natural logarithm.

The SIEM-2 contains Eqs. (1) (see above), (4), (5) as follows:

$$\ln((115.0500/N_{EH}) \times 100) = 6.4714^{a} + 0.1204 \times \times \ln isPA - 0.6349^{b} \times \ln N_{IN}, \quad (4)$$

$$\ln((34.4050/N_{IN})) = 6.9292^{a} - 0.7673^{a} \times \ln isPA, \quad (5)$$

where ^{a}, ^{b} – $p \leqslant 0.001, 0.01$.

The SIEM-3 contains Eqs. (1) (see above), (6), (7) as follows:

$$\ln((21.045/isPA) \times 100) = 7.4790^{a} - - 1.4494^{a} \times \ln N_{IN} + 0.4825 \times \ln N_{EH}, \quad (6)$$

$$\ln((34.4050/N_{IN}) \times 100) = 13.3878^{a} - 1.8373^{a} \times \ln N_{EH}, \quad (7)$$

where ^{a} is $p \leqslant 0.001$.

The SIEM-4 contains Eqs. (1) and (6) (see above), (8) as follows:

$$\ln((115.0500/N_{EH}) \times 100) = 6.3158^{a} - 0.4887^{a} \times \ln N_{IN}, \quad (8)$$

where ^{a} is $p \leqslant 0.001$.

The group of independent Eqs. (IEs) (9), (10) is as follows:

$$\ln((33.5950/ivUA) \times 100) = 8.0052^{a} - - 1.6055^{a} \times \ln(\ln^{2}(\ln^{2}(\ln^{2} C^{2}_{LH}))), \quad (9)$$

$$\ln((4.3750/C_{LH}) \times 100) = 8.2472^{a} - 0.6816^{c} \times \ln ivUA + + 0.0418^{b} \times \ln^{4} ivUA - 0.0099^{b} \times \ln^{5} ivUA, \quad (10)$$

where ^{a}, ^{b}, ^{c}, ^{d} – $p \leqslant 0.001, 0.01, 0.05, 0.1$.

The symbols GM, N_{IN}, N_{EH}, isPA, ivUA, C_{LH} denote the grain mass per sowing area, the content of inorganic N, easily hydrolysed N, the *in situ* proteolytic and *in vitro* urease activities, the labile humus C content, respectively. Eqs. (1)-(10) presented herein are variants of the linearised bounded exponential function denoted as log-ratio-Y_{max}; non-linearised versions of this function, on the one hand, asymptotically approach 0, and on the other hand, are limited to the maximum value of the dependent variable Y_{max} (the value in the numerator of the left-hand side of each Eq.). Depending on the grade b_i, this function can describe *i*) an exponential decline to 0; *ii*) an exponential increase, bounded from above by Y_{max} (in the case of an inverse relationship or a certain transformation of the variables). The bounded exponential function can be found, specifically, in the publications of T.C. Han *et al.* (2021), T. Kolokolnikov and D. Iron (2021), M.M.T. Lakshani *et al.* (2023); a non-linearised form of the same Eq. was presented by M. Portnoi *et al.* (2021).

SIEM-1, SIEM-2 characterise the mediation of isPA effect on GM by N_{IN}, N_{EH} traits in case of simultaneous action of these indicators of soil N status on the final biological productivity of winter wheat. SIEM-3, SIEM-4 reflect the mediator role of isPA in the additive effect of N_{IN}, N_{EH}, isPA on GM, provided that the effects of N_{IN} and N_{EH} on the specified trait of the plant production process are mediated by each other, and all soil N-indicators considered here simultaneously affect GM. IEs (9), (10) characterise the independent mathematical determinations of log-ratio-Y_{max} (ivUA) or log-ratio-Y_{max} (C_{LH}) by modified variables based on C_{LH} or ivUA, respectively.

Estimates of the statistical efficiency of SIEM-1-SIEM-4, IEs are presented in Table 4. For all Eqs. within SIEMs and IE (10), relatively low values of the regression standard error (RSE) were obtained (0.0332-0.0723), while for IE (9), the value of this characteristic was slightly greater (0.2745). The sizes of the coefficient of determination (R^2, the adjusted coefficient of determination ($R^2_{(Adj)}$) for all Eqs. in SIEMs and IEs were significant and amounted to 0.7838-0.9670, 0.7658-0.9610, respectively, while the statistical validity of the Fisher coefficient (FC = 40.2072-162.5170) was $p < 0.001$. The indicated ranges of the SIEMs and IEs estimates were accompanied by rather small values of the MAPE: 0.5612-2.8312 (Table 4). For all Eqs. within the SIEMs and IEs, the statistical reliabilities of the hypotheses H_1 "heteroscedasticity occurs" and H_1 "there is a deviation of the residuals from normality" were $p > 0.005$, reflecting the absence of heteroscedasticity and violations in the ND residuals. The determinants of the covariance matrices of the residuals for SIEM-1-SIEM-4 were quite close to 0; the corresponding ln-values were within −17.8911-−16.1084, providing compelling evidence of linear dependencies between the individual Eqs. in these SIEMs. For the IE (10), the ln-determinant of the covariance matrix of the regression coefficients was −43.7045, while for the IE (9) it was −8.0508. This may suggest that IE (10) is much more consistent with a linear relationship between the predictors and the response variable than IE (9).

***Table 4**. Statistical performance estimates of the SIEMs and IEs*

No. of of the SIEMs, IEs and Eqs.	***RSE***	***R^2***	***$R^2_{(Adj)}$***	***FC***	***MAPE***	***Heteroscedasticity test (HST)***	***Residue normality test (RNT)***	***Ln(det) of the cov-matrix^{e}***
SIEM-1								
1	0.0723	0.9499	0.9348	63.1363$^{\text{æ}}$	1.4285	1.0000$^{\text{ʃ}}$	0.2591$^{\text{ʒ}}$	-17.8911$^{\text{ʣ}}$
2	0.0377	0.9670	0.9610	161.1814$^{\text{æ}}$	0.6430			
3	0.0479	0.7997	0.7830	47.9096$^{\text{æ}}$	0.7758			
SIEM-2								

Table 4. Continued

No. of of the SIEMs, IEs and Eqs.	RSE	R^2	$R^2_{(Adj)}$	FC	MAPE	Heteroscedasticity test (HST)	Residue normality test (RNT)	Ln(det) of the cov-matrixɐ
1	-ɸ	-ɸ	-ɸ	-ɸ	1.4285	1.0000ɠ	0.2888ʒ	-17.8911ʣ
4	0.0332	0.9039	0.8864	51.7170æ	0.5612			
5	0.0544	0.9312	0.9255	162.5170æ	0.98755			
SIEM-3								
1	-ɸ	-ɸ	-ɸ	-ɸ	1.4285	1.0000ɠ	0.5302ʒ	-16.1084ʣ
6	0.0664	0.9352	0.9235	79.4192æ	1.1983			
7	0.0662	0.8980	0.8894	105.5880æ	1.1869			
SIEM-4								
1	-ɸ	-ɸ	-ɸ	-ɸ	1.4285	1.0000ɠ	0.5081ʒ	-17.4327ʣ
6	-ɸɸ	-ɸɸ	-ɸɸ	-ɸɸ	1.1983			
8	0.03416	0.8980	0.8894	105.5631æ	0.5742			
IE								
9	0.2745	0.7838	0.7658	43.5126æ	2.8312	0.4407–0.7132ɟ	0.7478ʤ	-8.0508ʤ
10	0.0335	0.9234	0.9005	40.2072æ	0.32962	0.5068–0.8590ɟ	0.7504ʤ	-43.7045ʤ

Note: *RSE, R^2, $R^2_{(Adj)}$, MAPE, FC – the regression standard error, the coefficient of determination, the adjusted coefficient of determination, the mean absolute percentage error, the Fisher coefficient, respectively; ɐ – the natural logarithm of the determinant of the covariance matrix; æ – the statistical reliability of the FC – $p < 0.001$; ɸ or ɸɸ – the same values (sizes) of efficiency estimates, as in the case of the previous Eq. (1), SIEM-1 or Eq. (6), SIEM 3; ɠ, ʒ – the sizes (quantities) of the statistical validity p of the hypothesis H_1 (HST (Breusch-Pagan)): "heteroscedasticity occurs", and H_1 (RNT (Doornik-Hansen)): "there is a deviation from normality"; ɟ, ʒ – the intervals of sizes (quantities) of the statistical validity p of the hypothesis H_1 (HST(White-only squares and the usual full versions, Breusch-Pagan regular and robust versions)): "heteroscedasticity occurs", and H_1 (RNT): "there is a deviation from normality", respectively; ɐ – the natural logarithm of the determinant of the covariance matrix of residuals of the SIEM or of the covariance matrix of the regression coefficients of each independent OLM-Eq; ʣ, ʤ – obtained from the covariance matrix of the residuals of different Eqs. of each SIEM or from the covariance matrix of the regression coefficients of each independent OLM-Eq, respectively*

Source: *developed by the authors of this study*

Considering that for Eqs. (2), (3), (4), (5) within SIEM-1, SIEM-2, the RSE values (0.0332-0.0544), the MAPE range (0.5612-0.98755), and the values of the ln-determinant of the covariance matrix of residuals (−17.8911) are smaller compared to the corresponding estimates for Eqs. (6), (7), (8) in SIEM-3, SIEM-4 (0.03416-0.0664; 0.5742-1.1983; −17.4327-−16.1084) (Table 4), it is reasonable to assume that SIEM-1, SIEM-2 may have a certain priority over SIEM-3, SIEM-4. However, the comparison of these Eqs. in the above groups of models by R^2, $R^2_{(Adj)}$ (0.7997-0.9670, 0.7830-0.9610 vs. 0.8980–0.9352, 0.8894-0.9235, respectively) (Table 4) rather testifies in favour of the advantages of SIEM-2 over the rest of the analysed SIEMs. Still, it is possible that SIEM-1 will become relevant if and when Eqs. (2), (3) still dominate Eqs. (4), (5) under experimental and/or real field conditions. Nevertheless, SIEM-3, SIEM-4 can make a lot of sense in situations where there are various factors, stochastic disturbances that will cause deviations from the ND residuals of the considered SIEMs (greater *p* values (lower χ^2) for the H_1 (residual normality test): "there is a deviation from normality" for SIEM-3, SIEM-4 are more difficult to reject than the corresponding smaller *p* values for SIEM-1, SIEM-2).

Significantly smaller (~ by an order of magnitude) RSE and MAPE, greater R^2, $R^2_{(Adj)}$, and substantially smaller values of the natural logarithm of the covariance matrix of the regression coefficients for IE (10), compared to IE (9) (Table 4) argue for the probable advantages of the log-ratio-Y_{max} (C_{LH}) dependence on ivUA over the log-ratio-Y_{max} (ivUA) dependence on C_{LH}, however, without specifying causality. This assumption is also supported by a comparison of the values of the log likelihood, Akaike, Schwartz, and Hennane-Queen criteria (30.0500, −52.3366 to −49.5438 for IE (10) versus −0.686762, 5.2552 to 6.6516 for IE (9), respectively). Therewith, for each of the regressors in IE (10), structural shifts were found by the Chow test (centre of the sample); the latter may reflect a partially imperfect model specification and, possibly, the effects of multicollinearity on the properties of this model.

Notably, the 3SLS method is typically more consistent and provides more asymptotic estimates for SIEM and SEM compared to 2SLS, OLS. This is not least conditioned by the fact that 3SLS is a combination of 2SLS and SUR (Tetteh, 2024). In this regard, 3SLS jointly estimates the entire model in a structural form and factors in the simultaneous cross-correlations between errors

(Butkus *et al*., 2024). Such modelling properties allow adjusting the weighting matrix for potential heteroscedasticity problems in 3SLS regression equations using the covariance matrix of the residuals from 2SLS; the generalised least squares method is usually used for parameter estimation (Huda & Istiana, 2024). However, 2SLS is effective when there is a secondary predictor that is correlated with the problem predictor (even hypothetically) but not with the error term. In the case when stochastic disturbances in different equations are uncorrelated or there is no heteroscedasticity, the results of 2SLS are typically equivalent to 3SLS estimates (Tetteh, 2024). Notably, the advantages of 2SLS over 3SLS may substantially depend on the level of multicollinearity between the variables (Oduntan, 2023). It would not be unreasonable to assume that the priority of 2SLS, or even OLS *vs*. 3SLS, at least in part, may also be conditioned by contamination of variables by errors, specifically, with small samples, depending on the accuracy of identification of Eqs., the type of distribution, and sample size. Overall, it is possible that the combination of similar multicollinearity and identification problems, in addition to small sample sizes, were significant reasons for the preference of 2SLS (SIEM-1 – SIEM-4) and OLS (IEs (9), (10)) over 3SLS.

Therewith, the approach to formulating and evaluating SEM, SIEM (even if it is 2SLS) can be considered as a task for an artificial neural network (ANN), and therefore it makes sense to consider SEM, SIEM as special cases of ANN (Pérez-Sánchez *et al*., 2024). The latter clearly suggests the relevance of using SEM, SIEM to solve various problems, at least not of a large size, based on mathematical and statistical regularities and assumptions about causality between variables. It is natural to assume that the mentioned significance of SEM, SIEM is inherent in biological and agricultural scientific problems.

DISCUSSION

The authors of the present study assumed that changes in N_{IN}, N_{EH}, C_{LH} in the soil of variants A-1 – B-7 are the outcome of a complex interaction of physicochemical and biological processes initiated by plant residues (PR – FPS/FBS), mineral fertilisers (RMF/HMF) and/or bioeffectors (BS/HF/CF). Considering the following arguments, it is worth noting that the C:N ratio in biomass or straw (STR) of legumes is several times lower than in non-legumes (Watthier *et al*., 2020; Grimm *et al*., 2024) and may approach the threshold of 25:1 for this criterion in the substrate (S) (Cao *et al*., 2021). Thus, it is likely that the addition of legume STRs alone (variants A-2, B-2) stimulated N-releasing microbial activity (MA), N-cycling, and gross mineralisation (Robertson & Groffman, 2024), providing energy for heterotrophic PR-degrading microflora (analogous to priming – PRM (Jesmin *et al*, 2021)), although efficiency may be reduced under N deficiency due to N sequestration in the microbial biomass (MB) (Henneron *et al*., 2020).

Reducing C:N in OM or adding N (also possibly N_{IN}) can enhance net N mineralisation and weaken N immobilisation (Sawicka *et al*., 2020; Cao *et al*., 2021; Robertson & Groffman, 2024), which is known as PRM (Meng *et al*., 2020) and was observed in A-3, B-3 (FPS/FBS + RMF). Such intensive treatments can increase PR and CO_2-production, but do not promote C-sequestration (Jesmin *et al*., 2021). Mineralisation of OC (especially under N-stimulation) releases C,N-S to MB, enhancing MA, which decomposes PR, increasing $N\text{-}NH_4^+$ and $N\text{-}NO_3^-$ (observed for N_{IN} in A-3, B-3). The formation of N_{IN} is preceded by the formation of available SON from its turnover in soil and PR (Yan *et al*., 2020). The evident partial overlap of the fraction of dissolved organic nitrogen (DON) with N_{EH} explains the unidirectional changes in N_{IN} and N_{EH} in the soil of the studied variants (Farzadfar *et al*., 2021). Comparable changes in C_{LH} in A-3, B-3 are probably conditioned by the decomposition of SOM and SON, especially with PR.

The increase in the *in situ* PA (isPA) in A-2 and B-2, compared to A-1, B-1, was consistent with stimulated mineralisation due to a decrease in C:N in S and accumulation of N_{IN}, N_{EH}, C_{LH} (Jesmin *et al*., 2021). Removal of N-limiting (N-LIM) can increase microbial attack on fresh OM (Henneron *et al*., 2020). The levelling of C-LIM by available N (Luo *et al*., 2021) explains the analogous changes in isPA, N_{IN}, N_{EH}, C_{LH} in A-3, B-3, relative to A-1, B-1, which is also consistent with the dependence of protein mineralisation on S and proteases (Greenfield *et al*., 2020) and the correlation of PA with DON (Farzadfar *et al*., 2021), and thus probably with N_{EH}. Unidirectional changes of the *in vitro* UA (ivUA) and isPA (at least in A-1 – B-3) are predicted, specifically, due to the participation of UA in N cycling and based on the findings of J. Yang *et al*. (2020), M.I. Khan *et al*. Experiments by P. Tian *et al*. (2020) revealed the effect of STR ploughing depth on ivUA. The application of N fertiliser + rice STR increased ivUA (Sharma *et al*., 2021), while commercial mulch decreased ivUA in long-term experiments (Sun *et al*., 2021).

Changes in soil physicochemical properties in A-4 – B-7 are potentially caused by a combination of PRM (N-stimulated mineralisation of crop residues, as in A-3, B-3) and rhizosphere PRM (rPRM), considering the significance of the relationship between the rhizosphere and bulk soil (Sun *et al*., 2021). In A-4 – A-6/B-4 – B-6, rPRM could be initiated with BS, BS+HF or CF. BSs contain phytohormones and C,N-auxiliary compounds that stimulate metabolism, growth, and nutrient uptake (Franzoni *et al*., 2022), and CFs may act similarly to BSs and HFs (Musych *et al*., 2024). It was assumed that BS, BS+HF, CF stimulated N uptake, photosynthesis, wheat growth, and C-rhizodeposits release, which could enhance N cycling through rPRM degradation of SOM (Henneron *et al*., 2020). This could have been combined with bulk soil N-PRM (A-3, B-3), causing an increase in N_{IN}, N_{EH}, C_{LH}, NME. However, the absence of substantial differences in N_{IN}, N_{EH}, C_{LH}, isPA, ivUA in A-4, A-6, B-4, B-6 *vs.* A-3, B-3 reflects insignificant synergistic effects between the 2 types of PRM and, probably, the predominance of the influence of N-stimulated PR mineralisation. The power and duration of these technologies are clearly insufficient for C and N sequestration.

Conversely, A-5, B-5 had maximal C_{LH}, insignificant changes in N_{IN}, N_{EH}, isPA, and a significant increase in ivUA (the latter complicates the assessment of N sequestration) compared to A-3, B-3. It was assumed that the synergistic effect of BS+HF in A-5, B-5 stimulated the rhizosphere turnover of SOM, probably activating C-cycles throughout the soil. Q. Ma *et al.* (2020) argued that N mineralisation and nitrification increase with decreasing N- and increasing C-LIMs Therefore, the decrease in C-LIMs due to increased SOM turnover led to the accumulation of C_{LH} in A-5, B-5, while the intensity of N-turnover and N_{IN}, N_{EH}, isPA were comparable to A-3, A-4, A-6 and B-3, B-4, B-6. Based on the hypothetical pattern of C sequestration under chronic N deposition (Lu *et al.*, 2021) and the findings of Q. Ma *et al.* (2020), partial enhancement of C- and weakening of N-LIMs in A-7, B-7 (HMF-apl.) was assumed. The rPRM from HF may not generally compensate for C-LIM in these variants, which was accompanied by a moderate decrease in C_{LH} turnover, but also a moderate increase in N mineralisation (N_{IN}, N_{EH}, isPA); ivUA co-changed with C_{LH}.

Changes in N, C, and NME forms reflect the functioning and interaction of microbes and plants at the soil level, where intermediates (IM) and final products (FP) of these cycles are a source of plastic material and energy. MA, particularly NME, can influence crop productivity, which is consistent with the observed correlation (PMC) (N_{IN}, N_{EH}, C_{LH}) – GM, as well as (isPA, ivUA) – GM. The significance of soil NME for plant productivity was confirmed by studies (Zhou *et al.*, 2020; Yang *et al.*, 2022; Garnaik *et al.*, 2022), which showed $r > 0$ for (ivPA/UA) – (wheat/rice yield). The absence of GM dependence on isPA within the GLM used may be partially consistent with the findings of other studies (Nath *et al.*, 2021), where no significant single-factor dependence (rice yield) – ivPA was found. However, the authors of the present study believe that the results of the incompletely parameterised GLM provide insufficient information to understand the causal relationships between the considered regressors and GM.

The EFA results reflect that GM is dependent on N_{IN}, N_{EH}, and isPA (factor 1, FA-1), while C_{LH} and ivUA form a separate FA-2, potentially affecting each other but not directly GM, which may reflect the partial decoupling of the N and C cycles and the asynchrony of N cycle elements (Ma *et al.*, 2020). The N-LIM of microorganisms may undergo a deepening under excessive available C (PR), which will stimulate compensatory N-mineralisation (Tan *et al.*, 2021; Qiu *et al.*, 2021; Cui *et al.*, 2022), explaining the spatial and temporal divergence of the N and C cycles. Admittedly, with the depletion of labile fractions of plant SOM/SON, microbes attack N-enriched SOM, activating N-mineralisation, which increases N availability. Conversely, the presence of C in low-energy compounds favours C-LIM. Thus, changes in the energy supply of microorganisms, affecting the C/N balance, limit N mineralisation and break the causality between microbial C and N availability/demand (Zhang *et al.*, 2021).

The link between C_{LH} and ivUA may be mediated by intensification of ureolysis, which increases N availability, promoting the development of microbial communities less capable of degrading recalcitrant C, leading to C sequestration and reduced NME activity. Indirect C-input from plant residues can stimulate EA, particularly UA (Tan *et al.*, 2021). Other studies have not found a direct correlation between rice/maize yields and ivUA (Nath *et al.*, 2021; Kalala *et al.*, 2022). The distribution of variables by FA-1, FA-2 partially revises the hypotheses about the causal relationships between soil characteristics and wheat GM. In method (*i*) (abbreviated: NME → agrochemical/physical-chemical soil characteristics → GM), the simultaneous additive effect of N_{IN}, N_{EH}, isPA on GM is assumed (Eq. (1)), where each of the pools of N_{IN}, N_{EH} depends on isPA and another pool (Eqs. (2), (3) in (SIEM-1, Eqs. (4), (5) in SIEM-2). Thus, the effect of isPA on GM can be mediated through N_{IN} (depending on the ratio of isPA and N_{EH}) or through N_{EH} (depending on the ratio of isPA and N_{IN}).

To argue for a positive feedback loop (PFL) (method (*ii*); abbreviated: agrochemical/physical-chemical soil characteristics → NME activity → GM), the resource allocation theory is applied, according to which nutrients-LIM for microorganisms stimulates the production of eco-EA (EEA) for their assimilation (Luo *et al,* 2021; Mori *et al.*, 2021), i.e., EEA ∝ 1/(nutrient availability) (negative feedback loop (NFL), possible under C-LIM; L. Luo *et al.* (2021)). C-enrichment can limit microorganisms, causing an increase in EA for other nutrients, particularly N (Tan *et al.*, 2021), which could account for the PFL between EA and N and C forms found by P. Tian *et al.* (2020). Enhanced N mineralisation can stimulate NME more strongly than EA to produce C (Qiu *et al.*, 2021). The size of the hydrolase pool (V_{max}) ∝ of the S concentration (Tan *et al.*, 2021). Causality method (*ii*) assumes a combination of simultaneous determination of GM by N_{IN}, N_{EH}, isPA with the predetermination of isPA by the ratio N_{EH}/N_{IN} and mutual determination of N_{IN} and N_{EH}. Hence, Eq. (1) is combined with the mediation of the effect of the N_{IN}/N_{EH} balance through isPA on GM and the mediation of the effect of N_{EH} on GM through N_{IN} (or vice versa) (SIEM-3/SIEM-4). In causality mode (*ii*), GM simultaneously depends on N_{IN}, N_{EH}, and isPA (Eq. (1)), while isPA is mediated by N_{EH}/N_{IN} (Eq. (6)), while N_{IN} and N_{EH} are mutually determined (Eqs. (7), or (8); SIEM-3/SIEM-4).

Eq. (1) showed the direct effect of N_{IN} and N_{EH} on GM growth and the inverse dependence of GM on isPA.

In SIEM-1, an increase in isPA and N_{EH} mediated an increase in N_{IN}, while an increase in isPA alone mediated an increase in N_{EH}. In SIEM-2, a decrease in isPA and an increase in N_{IN} could mediate the effect of N_{EH}, while an increase in isPA alone could cause an increase in N_{IN}. In SIEM-3/SIEM-4, an increase in N_{IN} and a decrease in N_{EH} were influenced by an increase in isPA, while an increase in N_{EH} caused an increase in N_{IN} (and vice versa). All these dependencies are statistically significant. The statistical significance of the relationship between C_{LH} and ivUA (IE 10) reflects correlation rather than causation, and C_{LH} may be influenced by other factors. The expected positive effect of N_{IN} and N_{EH} on GM (Eq. (1)) contrasts with the unexplained inverse relationship between isPA and GM. This contradiction is partially explained by the dependence of N_{IN} growth on isPA and N_{EH}, and N_{EH} growth on isPA (Eqs. (2), (3)), as well as the dependence of N_{EH} growth on isPA decline and N_{IN} growth, and N_{IN} growth on isPA increase (Eqs. (4), (5)).

The positive effect of N_{EH} and isPA on N_{IN} (Eq. (2)) suggests that protease-mediated mineralisation of S from N_{EH} (isPA) increases the available forms of N_{IN}, which is consistent with the concept of N-mineralisation (Robertson & Groffman, 2024) and the significance of SON/DON (Ma *et al.*, 2020; Yan *et al.*, 2020). This is also consistent with the role of soil TN (and likely SON) and MBN in controlling nitrification (Li *et al.*, 2020). The ability of isPA to stimulate N_{IN} accumulation (Eq. (2)) supports the concept that protein mineralisation rates depend on available S and protease production (Greenfield *et al.*, 2020), with the amount of S (N_{EH}) being more significant. Interpreting N_{EH} as proteolysis products, its level increases with increasing isPA (Eq. (3)), which is also consistent with N-mineralisation (depolymerisation of SON by isPA leads to the accumulation of N_{EH}/DON; X. Sun *et al.*, 2021) and the effect of ivPA and other factors on SON (Yang *et al.*, 2020). The triggering effect of isPA on N_{EH} (Eq. (3)) and the consolidated effect of isPA and N_{EH} on N_{IN} (Eq. (2)) are manifested in Eq. (1) as additive stimulating effects of N_{IN} and N_{EH} on wheat GM with simultaneous suppressive effect of isPA on GM. It is unclear whether this is a direct effect of isPA on GM or a latent NFL between this EA and N_{IN}/N_{EH}, which is consistent with the theory of resource allocation in the soil.

In SIEM-2, the effect of isPA on N_{IN} and N_{EH} with the stimulating effect of the latter on GM and the NFL from isPA on GM are complex. The key is the activating effect of isPA on N_{IN} (Eq. (5)), simultaneously with the suppressive effect of isPA and the stimulating effect of N_{IN} on N_{EH} (Eq. (4)). The ability of isPA to increase N_{IN} (Eq. (5)) is consistent with the characteristics of N-mineralisation and proteolysis. The stimulatory effect of N_{IN} on N_{EH} (Eq. (4)) may be conditioned by PFL, which is consistent with the significance of SON turnover (Yan *et al.*, 2020) and soil N priority (Anas *et al.*, 2020), and is also consistent with FL control by N_{IN} of the BAN pool (Daly *et al.*, 2021). Enhanced microbial C supply and activation of N-mineralisation could contribute to the development of PFL control of NME/isPA by N_{IN}. The stimulatory effect of N_{IN} on N_{EH} (as a product of isPA) may reflect the stimulation of isPA levels by $N_{IN,}$ which is partially consistent with the findings of L. Greenfield *et al.* (2020). The inverse effect of isPA on N_{EH} may reflect a decrease in S with an increase in EA. Therefore, in SIEM-2, isPA stimulates N_{IN} (Eq. (5)), which increases N_{EH} (Eq. (4)), and the growth of both forms of N positively influences GM (Eq. (1)). The simultaneous increase of isPA balances the activating effect of N_{IN} on N_{EH} (Eq. (4)) and counteracts the favourable effects of these forms of N on GM (Eq. (1)).

For SIEM-3 and SIEM-4, the feedback between isPA and its products is significant. The stimulatory effect of N_{IN} on N_{EH} (Eq. (8), SIEM-4, as in Eq. (4), SIEM-2) may reflect the enhancement of isPA by FP proteolysis and N-mineralisation, which is consistent with the activating effect of N_{IN} on isPA (Eq. (6), SIEM-3/4). The direct relationship between N_{IN} and N_{EH} (Eq. (7), SIEM-3) reflects that N_{IN} increases with increasing N_{EH}. The inhibitory effect of N_{EH} (as products of isPA) on isPA (Eq. (6), SIEM-3/4) indicates FL. Therefore, in SIEM-3/4, an increase in N_{EH} (the result of isPA) causes an increase in N_{IN} (Eq. (7)), which can enhance isPA (Eq. (6)) and expand N_{EH} (Eq. (8)), while an increase in N_{EH} suppresses isPA. The indirect ratio of the effects of N_{IN} and N_{EH} on isPA increase can cause GM falloff, while the consolidated gain of these N forms is GM gain (Eq. (1)). The stimulatory effect of N_{IN} on N_{EH} can lead to an indirect decrease in GM gain by isPA; on the other hand, the expansion of N_{IN} can cause an increase in isPA, which will lead to GM falloff.

This feedback is different from the one proposed by X. Tan *et al.* (2021). However, the stimulatory effect of N_{IN} on N_{EH} and isPA (Eqs. (4), (8), (6)) is consistent with the ability of N_{IN} to increase ivPA, which is consistent with $r > 0$ for $N\text{-}NH_4^+$ – ivPA (Sawicka *et al.*, 2020), but partially contradicts the $r < 0$ found by them for $N\text{-}NO_3^-$ – ivPA. These interpretations may be opposed by the effects of ivPA inhibition by high concentrations of NH_4^+ or NO_3^- (Jesmin *et al*, 2021) and a moderate $r < 0$ between the rate of slow phase protein mineralisation and NO_3^- (Greenfield *et al.*, 2020), although a possible $r > 0$ between N-cycle enzyme (NAG) activity and NO_3^- (Błońska *et al.*, 2021). The attenuation of isPA by N_{EH} is partially consistent with the resource allocation theory and the concept of decreasing NME activity with increasing available N forms (He *et al.*, 2021), where the size of the N_{EH} pool may reflect the availability of N. Additionally, E. Błońska *et al.* (2021) argued that mineralisation dominates immobilisation in the decomposition of N-rich detritus, which may lead to a decrease in NME production and $r < 0$ between this EA and SON/DON/N_{EH} components, but $r > 0$ between NAG and N light fractions is possible (Błońska *et al.*, 2020).

The statistical efficiency of IE (10) versus IE (9) and the greater nonlinearity of the latter reflect a probable dependence of C_{LH} on ivUA, rather than vice versa.

However, the activating effect of humic acids (HA) and vermicompost on ivUA is known to be accompanied by an increase in microbial population and C:N, while HA from weathered coal/leonardite can inhibit ivUA (Ampong *et al.*, 2022). IvUA is nonlinearly dependent on the mass fraction of soil macroaggregates (Zhao *et al.*, 2020), but the effect of initial OM and MBC on ivUA, as well as the dependence of SON on the latter, was not confirmed (Yang *et al.*, 2020). At the same time, an inverse relationship between ivUA and MBC was found (Sun et al., 2021). There is evidence of the effect of ivUA on SOC components, including a significant effect on fine POC (Liu *et al.*, 2021) and participation in the pre-mortemisation of other SOC fractions. IvUA can mediate the effect of organic fertiliser on MBC and its effect on LOC, as well as the effect of organic fertiliser on LOC/UOC and their effect on SOC (Zhang *et al.*, 2022).

The obtained results of EFA, SIEMs, IEs do not show a clear direction of changes in ivUA and N conversion/mineralisation, unlike the study of organic tree mulching (Sun *et al.*, 2021). However, the PMC between ivUA – N_{IN} is partially consistent with the directly proportional ivUA – NH_4^+ relationships (Sun *et al.*, 2021; Sawicka *et al.*, 2020), the high probability of ivUA – $N_{IN}/NH_4^+/NO_3^-$ relationships (Sharma *et al.*, 2021), and the positive effect of NO_3^- on ivUA (Tian *et al.*, 2020). Inconsistencies between correlation and multivariate analyses, as well as ambiguities in their agreement with the literature, may be in favour of the expected spatial and temporal divergence of N and C cycle trajectories in all tested variants, which may be conditioned by *i*) the specific features of N- and C-mineralisation, PRM and the role of plants in each variant, *ii*) insufficient number of explanatory variables to reflect the relationships between N- and C-cycles in soil in relation to plant productivity. The application of PR with/without fertilisers and bioeffectors determined the direction of N- and C-mineralisation, where variations in N_{IN}, N_{EH}, isPA were the probable causes of GM changes. All the analysed causal relationships were characterised by a positive effect of N_{IN} and N_{EH} on GM and a possible suppressive effect of isPA on GM. The increase in GM due to an increase in N_{IN}/N_{EH} can potentially be accompanied by a decrease in isPA (a possible latent NFL), which is partially consistent with the theory of resource allocation and the stoichiometry of N-mineralisation.

Considering the statistical efficiency of SIEMs, mediation of isPA effect on GM via N_{IN}/N_{EH} (SIEM-1/2) is more probable than mediation of N-forms effect on GM via isPA (SIEM-3/4). The most plausible is the stimulating effect of N_{IN} on isPA and the dependence of N_{EH} on the balance between isPA and N_{IN} (SIEM-2). Less relevant are the dependence of N_{EH} on isPA and the conditionality of N_{IN} on the consolidated effect of isPA, N_{EH} (SIEM-1). Statistically, the least reliable isPA control of the N_{EH}/N_{IN} ratio (SIEM-3/4) may be consistent with latent NFL and may be updated under certain conditions. Simultaneously with the influence of N-characteristics on GM, there may be a separate dependence of C_{LH} on ivUA and other unidentified factors (IE 10), while the dependence of ivUA on C_{LH} (IE 9) is less likely. The reasons for the separation of C_{LH} and ivUA from the N-characteristics continue to be unknown but can hypothetically be related to the specific features of microbial and plant economics in various experimental variants.

CONCLUSIONS

The expediency of argumentation of changes in N-, C-chemical, N-biological soil indicators (N_{IN}, N_{EH}, C_{LH}, *in situ* PA – isPA, *in vitro* UA – ivUA) in the context of the following hypothetical and theoretically predictable processes in soil is illustrated: (*i*) N-, C-mineralisation/immobilisation, (*ii)* sequestration, (*iii*) priming, (*vi*) aspects of resource allocation. The most effective were the rationales built in the areas (*i*), (*iii*), (*vi*). The statistical validity of the predetermination of grain weight per sown area of GM winter wheat by the interdependencies N_{IN}, N_{EH}, isPA was proved. The simultaneous significant probability of the interrelationships between C_{LH} and ivUA revealed the disconnect between the availability and demand of microorganisms for C and N under these experimental conditions. Among the crucial properties of the causal relationships between these variables were the stimulating effect of N_{IN}, $N_{EH,}$ and the reverse effect of isPA on GM, as well as a more significant mediation of isPA action through N_{IN}, N_{EH} (especially when N_{IN} is predetermined by isPA level, and at the same time, N_{EH} size is predetermined by the ratio of isPA and N_{IN}), compared to the mediation of the proportion of N_{IN}, N_{EH} effects through isPA (considering the predetermination of N_{IN} by N_{EH} size or *vice versa*). In contrast to the first of the above variants of variable premixing, the second helped to partially isolate the elements of the basis for the opposite influence of N_{IN}, N_{EH}, on the one hand, and isPA, on the other hand, on GM formation. The non-causal conditioning of C_{LH} by ivUA did not take part in the development of GM variation but was probably associated with the effect of unidentified factor variables. The complex structure of the data (deviations from normal distribution, multicollinearity, etc.) and the presence of unidentified predictors herein, hinder the search for reliable simple, yet extensive mathematical formalisations for causality or directionality of relationships between variables. Prioritising combinations of N-, C-chemical, and biological soil indicators, identifying and analysing potential and/or actual causal relationships between them and crop productivity traits is vital for constructing minimum sets of parameters and indices of soil health and quality relevant for predicting potential and actual soil fertility. In the future, it is significant to search for and analyse, using statistical/machine learning methods, supplementary features to those presented herein that may contribute to the formation of the C/N balance in soil and crop bioproductivity, particularly under sustainable farming technologies.

ACKNOWLEDGEMENTS

None.

FUNDING

This study received no funding.

CONFLICT OF INTEREST

The authors of this study declare no conflict of interest.

REFERENCES

[1] Abdelwahed, S., Trabelsi E., Saadouli, I., Kouidhi, S., Masmoudi, A.S., Cherif, A., Mnif, W., & Mosbah, A. (2021). A new pioneer colorimetric micro-plate method for the estimation of ammonia production by plant growth promoting rhizobacteria (PGPR). *Main Group Chemistry*, 21(1), 55-68. doi: 10.3233/MGC-210077.

[2] Adamczyk, B. (2021). Root-derived proteases as a plant tool to access soil organic nitrogen; Current stage of knowledge and controversies. *Plants*, 10(4), article number 731. doi: 10.3390/plants10040731.

[3] Ampong, K., Thilakaranthna, M.S., & Gorim, L.Y. (2022). Understanding the role of humic acids on crop performance and soil health. *Frontiers in Agronomy*, 4, article number 848621. doi: 10.3389/fagro.2022.848621.

[4] Anas, M., Liao, F., Verma, K.K., Sarwar, M.A., Mahmood, A., Chen, Z.L., Li, Q., Zeng, X.P., Liu, Y., & Li, Y.R. (2020). Fate of nitrogen in agriculture and environment: Agronomic, eco-physiological and molecular approaches to improve nitrogen use efficiency. *Biological Research*, 53, article nammber 47. doi: 10.1186/s40659-020-00312-4.

[5] Blonska, E., Lasota, J., da Silva, G.R.V., Vanguelova, E., Ashwood, F., Tibbett, M., Watts, K., & Lukac, M. (2020). Soil organic matter stabilization and carbon-cycling enzyme activity are affected by land management. *Annals of Forest Research*, 63(1), 71-86. doi: 10.15287/afr.2019.1837.

[6] Brown, S., Tauler, R., & Walczak, B. (2020). *Comprehensive chemometrics: Chemical and biochemical data analysis* (2nd ed.). Amsterdam: Elsevier.

[7] Butkus, M., Dargenytė-Kacilevičienė, L., Matuzevičiūtė, K., Ruplienė, D., & Šeputienė, J. (2024). Simultaneous, endogenous, and bi-directional relationship between output, labour productivity, and employment: A 3SLS estimation of the employment version of Okun equation. *Journal of Business Economics and Management*, 25(5), 1030-1051. doi: 10.3846/jbem.2024.22425.

[8] Cao, Y., He, Z., Zhu, T., & Zhao, F. (2021). Organic-C quality as a key driver of microbial nitrogen immobilization in soil: A meta-analysis. *Geoderma*, 383, article number 114784. doi: 10.1016/j.geoderma.2020.114784.

[9] Cheng, C., Han, H., Wang, Y., He, L., & Sheng, X. (2020). Metal-immobilizing and urease-producing bacteria increase the biomass and reduce metal accumulation in potato tubers under field conditions. *Ecotoxicology and Environmental Safety*, 203, article number 111017. doi: 10.1016/j.ecoenv.2020.111017.

[10] Chettri, D., Sharma, B., Verma, A.K., & Verma, A.K. (2021). Significance of microbial enzyme activities in agriculture. In R. Soni, D.C. Suyal, P. Bhargava & R. Goel (Eds.), *Microbiological activity for soil and plant health management* (pp. 351-373). Singapore: Springer. doi: 10.1007/978-981-16-2922-8_15.

[11] Convention on Biological Diversity. (1992, June). Retrieved from https://zakon.rada.gov.ua/laws/show/995_030#Text.

[12] Convention on International Trade in Endangered Species of Wild Fauna and Flora. (1979, June). Retrieved from https://zakon.rada.gov.ua/laws/show/995_129#Text.

[13] Cui, J., Zhu, R., Wang, X., Xu, X., Ai, C., He, P., Liang, G., Zhou, W., & Zhu, P. (2022). Effect of high soil C/N ratio and nitrogen limitation caused by the long-term combined organic-inorganic fertilization on the soil microbial community structure and its dominated SOC decomposition. *Journal of Environmental Management*, 303, article number 114155. doi: 10.1016/j.jenvman.2021.114155.

[14] Dalias, P., & Christou, A. (2022). Secondary immobilization as a phase of N mineralization dynamics of soil organic inputs. *Nitrogen*, 3(4), 600-607. doi: 10.3390/nitrogen3040039.

[15] Daly, A.B., Jilling, A., Bowles, T.M., Buchkowski, R.W., Frey, S.D., Kallenbach, C.M., Keiluweit, M., Mooshammer, M., Schimel, J.P., & Grandy, A.S. (2021). A holistic framework integrating plant-microbe-mineral regulation of soil bioavailable nitrogen. *Biogeochemistry*, 154, 211-229. doi: 10.1007/s10533-021-00793-9.

[16] Daunoras, J., Kačergius, A., & Gudiukaitė, R. (2024). Role of soil microbiota enzymes in soil health and activity changes depending on climate change and the type of soil ecosystem. *Biology*, 13(2), article number 85. doi: 10.3390/biology13020085.

[17] DSTU 4725:2007. (2008). *Soil quality. Determination of the activity of potassium, ammonium, nitrate and chlorine ions by potentiometric method*. Retrieved from https://online.budstandart.com/ua/catalog/doc-page?id_doc=72424.

[18] DSTU 4729:2007. (2008). *Soil quality. Determination of nitrate and ammonium nitrogen in modification of NSC ISSAR named for O. N. Sokolovski*. Retrieved from https://online.budstandart.com/ua/catalog/doc-page?id_doc=72836.

[19] DSTU 4732:2007. (2008). *Soil quality. Methods for determining an available (labile) organic matter.* Retrieved from https://online.budstandart.com/ua/catalog/doc-page.html?id_doc=72886/.

[20] DSTU 7863:2015. (2016). *Soil quality. Determination of available hydrolyzable nitrogen by Kornfild method*. Retrieved from https://online.budstandart.com/ua/catalog/doc-page.html?id_doc=62745.

[21] DSTU ISO 11464-2001. (2003). *Soil quality. Pretreatment of samples for physico-chemical analyses*. Retrieved from https://online.budstandart.com/ua/catalog/doc-page?id_doc=52911.

[22] Dubytska, A.O., Kachmar, O.Y., Vavrynovych, O.V., & Dubytskyi, O.L. (2022). Biological processes in gray forest soil under winter wheat by ecologized fertilizing systems. *Foothill and Mountain Agriculture and Stockbreeding*, 71(1), 21-35. doi: 10.32636/01308521.2022-(71)-1-2.

[23] Farzadfar, S., Knight, J.D., & Congreves, K.A. (2021). Soil organic nitrogen: An overlooked but potentially significant contribution to crop nutrition. *Plant and Soil*, 462, 7-23. doi: 10.1007/s11104-021-04860-w.

[24] Fierer, N., Wood, S.A., & de Mesquita, C.P.B. (2021). How microbes can, and cannot, be used to assess soil health. *Soil Biology and Biochemistry*, 153, article number 108111. doi: 10.1016/j.soilbio.2020.108111.

[25] Franzoni, G., Cocetta, G., Prinsi, B., Ferrante, A., & Espen, L. (2022). Biostimulants on crops: their impact under abiotic stress conditions. *Horticulturae*, 8(3), article number 189. doi: 10.3390/horticulturae8030189.

[26] Garnaik, S., *et al*. (2022). Untangling the effect of soil quality on rice productivity under a 16-years long-term fertilizer experiment using conditional random forest. *Computers and Electronics in Agriculture*, 197, article number 106965. doi: 10.1016/j.compag.2022.106965.

[27] Greenfield, L.M., Hill, P.W., Seaton, F.M., Paterson, E., Baggs, E.M., & Jones, D.L. (2020). Is soluble protein mineralisation and protease activity in soil regulated by supply or demand? *Soil Biology and Biochemistry*, 150, article number 108007. doi: 10.1016/j.soilbio.2020.108007.

[28] Grimm, D., Sonntag, E., & Rahmann, G. (2024). Oyster mushroom cultivation on cereal and legume straw of poor feed quality. *Studies in Fungi*, 9(1), article number e010. doi: 10.48130/sif-0024-0010.

[29] Han, T.C., Yan, H., & Fu, L.Y. (2021). A quantitative interpretation of the saturation exponent in Archie's equations. *Petroleum Science*, 18, 444-449. doi: 10.1007/s12182-021-00547-0.

[30] Hassan, A., Fauziah, S.H., Auta, H.S., Pariatamby, A., Ossai, I.C., Barasarathi, J., & Aziz, A.A. (2022). Microbial enzymes: Role in soil fertility. In N.R. Maddela, A.S. Abiodun & R. Prasad (Eds.), *Ecological interplays in microbial enzymology* (pp. 155-187). Singapore: Springer. doi: 10.1007/978-981-19-0155-3_9.

[31] He, W., Zhang, M., Jin, G., Sui, X., Zhang, T., & Song, F. (2021). Effects of nitrogen deposition on nitrogen-mineralizing enzyme activity and soil microbial community structure in a Korean pine plantation. *Microbial Ecology*, 81, 410-424. doi: 10.1007/s00248-020-01595-6.

[32] Henneron, L., Kardol, P., Wardle, D.A., Cros, C., & Fontaine, S. (2020). Rhizosphere control of soil nitrogen cycling: A key component of plant economic strategies. *New Phytologist*, 228(4), 1269-1282. doi: 10.1111/nph.16760.

[33] Huda, Q., & Istiana, N. (2024). Economic globalization, economic growth, and human capital: Empirical evidence using three stage least square in Indonesia. *Barekeng: Journal of Mathematics and Its Applications*, 18(3), 1483-1496. doi: 10.30598/barekengvol18iss3pp1483-1496.

[34] Jesmin, T., Mitchell, D.T., & Mulvaney, R.L. (2021). Short-term effect of nitrogen fertilization on carbon mineralization during corn residue decomposition in soil. *Nitrogen*, 2(4), 444-460. doi: 10.3390/nitrogen2040030.

[35] Kalala, D.M., Shitumbanuma, V., Chishala, B.H., Mweetwa, A.M., & Fliessbach, A. (2022). Influence of soil fertility management on nitrogen mineralization, urease activity and maize yield. *Journal of Agricultural Science*, 14(2), 9-23. doi: 10.5539/jas.v14n2p9.

[36] Khan, M.I., Gwon, H.S., Alam, M.A., Song, H.J., Das, S., & Kim, P.J. (2020). Short term effects of different green manure amendments on the composition of main microbial groups and microbial activity of a submerged rice cropping system. *Applied Soil Ecology*, 147, article number 103400. doi: 10.1016/j.apsoil.2019.103400.

[37] Kolokolnikov, T., & Iron, D. (2021). Law of mass action and saturation in SIR model with application to Coronavirus modelling. *Infectious Disease Modelling*, 6, 91-97. doi: 10.1016/j.idm.2020.11.002.

[38] Kyriazos, T., & Poga, M. (2023). Dealing with multicollinearity in factor analysis: The problem, detections, and solutions. *Open Journal of Statistics*, 13(3), 404-424. doi: 10.4236/ojs.2023.133020.

[39] Lakshani, M.M.T., Chamindu Deepagoda, T.K.K., Hamamoto, S., Elberling, B., Fu, W., Yang, T., Fan, J., Ma, X., Clough, T., Smits, K.M., Parameswaran, T.G., Sivakumar Babu, G.L., & Chanakya, H. (2023). A new exponential model for predicting soil gas diffusivity with varying degree of saturation. *Vadose Zone Journal*, 22(1), article number e20236. doi: 10.1002/vzj2.20236.

[40] Lehmann, J., Bossio, D.A., Kögel-Knabner, I., & Rillig, M.C. (2020). The concept and future prospects of soil health. *Nature Reviews Earth & Environment*, 1, 544-553. doi: 10.1038/s43017-020-0080-8.

[41] Levine, D.M., Stephan, D.F., & Szabat, K.A. (2021). *Statistics for managers using Microsoft Excel. Ninth Edition*. London: Pearson.

[42] Li, Z., Zeng, Z., Tian, D., Wang, J., Fu, Z., Zhang, F., Zhang, R., Chen, W., Luo, Y., & Niu, S. (2020). Global patterns and controlling factors of soil nitrification rate. *Global Change Biology*, 26(7), 4147-4157. doi: 10.1111/gcb.15119.

[43] Liu, M., Li, P., Liu, M., Wang, J., & Chang, Q. (2021). The trend of soil organic carbon fractions related to the successions of different vegetation types on the tableland of the Loess Plateau of China. *Journal of Soils and Sediments*, 21, 203-214. doi: 10.1007/s11368-020-02710-3.

[44] Lu, X., Vitousek, P.M., Mao, Q., Gilliam, F.S., Luo, Y., Turner, B.L., Zhou, G., & Mo, J. (2021). Nitrogen deposition accelerates soil carbon sequestration in tropical forests. *Proceedings of the National Academy of Sciences*, 118(16), article number e2020790118. doi: 10.1073/pnas.2020790118.

[45] Luo, L., Zhu, L., Hong, W., Gu, J.D., Shi, D., He, Y., Xiao, Y., Tian, D., Zhang, S., Deng, S., Lan, T., & Deng, O. (2021). Microbial resource limitation and regulation of soil carbon cycle in Zoige Plateau peatland soils. *Catena*, 205, article number 105478. doi: 10.1016/j.catena.2021.105478.

[46] Ma, Q., Wen, Y., Wang, D., Sun, X., Hill, P.W., Macdonald, A., Chadwick, D.R., Wu, L., & Jones, D.L. (2020). Farmyard manure applications stimulate soil carbon and nitrogen cycling by boosting microbial biomass rather than changing its community composition. *Soil Biology and Biochemistry*, 144, article number 107760. doi: 10.1016/j.soilbio.2020.107760.

[47] Magomya, A.M., Yebpella, G.G., Okpaegbe, U.C., Agu, M.O., & Kefas, K. (2022). An enzymatic method for the determination of urea in aqueous samples using crude urease from Citrullus lanatus seeds. *Journal of Chemical Society of Nigeria*, 47(4), 679-687. doi: 10.46602/jcsn.v47i4.777.

[48] Meng, X., Li, Y., Yao, H., Wang, J., Dai, F., Wu, Y., & Chapman, S. (2020). Nitrification and urease inhibitors improve rice nitrogen uptake and prevent denitrification in alkaline paddy soil. *Applied Soil Ecology*, 154, article number 103665. doi: 10.1016/j.apsoil.2020.103665.

[49] Mori, T., Aoyagi, R., Kitayama, K., & Mo, J. (2021). Does the ratio of β-1,4-glucosidase to β-1,4-*N*-acetylglucosaminidase indicate the relative resource allocation of soil microbes to C and N acquisition? *Soil Biology and Biochemistry*, 160, article number 108363. doi: 10.1016/j.soilbio.2021.108363.

[50] Musych, O.H., Zubko, O.V., & Dushko, P.M. (2024). Foliar nutrition of plants: Relevance, needs, implementation of tasks. *Agroecological Journal*, 2, 155-165. doi: 10.33730/2077-4893.2.2024.305675.

[51] Nath, C.P., Kumar, N., Das, K., Hazra, K.K., Praharaj, C.S., & Singh, N.P. (2021). Impact of variable tillage based residue management and legume based cropping for seven years on enzymes activity, soil quality index and crop productivity in rice ecology. *Environmental and Sustainability Indicators*, 10, article number 100107. doi: 10.1016/j.indic.2021.100107.

[52] Nathan, V.K., Vijayan, J., & Parvathi, A. (2020). Optimization of urease production by *Bacillus halodurans* PO15: A mangrove bacterium from Poovar mangroves, India. *Marine Life Science & Technology*, 2, 194-202. doi: 10.1007/s42995-020-00031-5.

[53] Oduntan, E. (2023). A Monte Carlo experiment on the asymptotic efficiency of system estimators under multicollinearity. *Journal of Statistical Modelling & Analytics*, 5(2). doi: 10.22452/josma.vol5no2.1.

[54] Pan, B., Xia, L., Lam, S.K., Wang, E., Zhang, Y., Mosier, A., & Chen, D. (2022). A global synthesis of soil denitrification: Driving factors and mitigation strategies. *Agriculture, Ecosystems & Environment*, 327, article number 107850. doi: 10.1016/j.agee.2021.107850.

[55] Pérez-Sánchez, B., Perea, C., Ballester, G.D., & López-Espín, J.J. (2024). Estimation of simultaneous equation models by backpropagation method using stochastic gradient descent. *PeerJ Computer Science*, 10, article number e2352. doi: 10.7717/peerj-cs.2352.

[56] Portnoi, M., Haigh, P.A., Macdonald, T.J., Ambroz, F., Parkin, I.P., Darwazeh, I., & Papakonstantinou, I. (2021). Bandwidth limits of luminescent solar concentrators as detectors in free-space optical communication systems. *Light: Science & Applications*, 10, article number 3. doi: 10.1038/s41377-020-00444-y.

[57] Qiu, X., Peng, D., Tian, H., Wang, H., Liu, X., Cao, L., Li, Z., & Cheng, S. (2021). Soil ecoenzymatic stoichiometry and microbial resource limitation driven by thinning practices and season types in *Larix principis-rupprechtii* plantations in North China. *Forest Ecology and Management*, 482, article number 118880. doi: 10.1016/j.foreco.2020.118880.

[58] Robertson, G.P., & Groffman, P.M. (2024). Nitrogen transformations. In E. Paul & S. Frey (Eds.), *Soil microbiology, ecology and biochemistry.* (pp. 407-438). Amsterdam: Elsevier. doi: 10.1016/B978-0-12-822941-5.00014-4.

[59] Sawicka, B., Krochmal-Marczak, B., Pszczółkowski, P., Bielińska, E.J., Wójcikowska-Kapusta, A., Barbaś, P., & Skiba, D. (2020). Effect of differentiated nitrogen fertilization on the enzymatic activity of the soil for sweet potato (*Ipomoea batatas* L.[Lam.]) cultivation. *Agronomy*, 10(12), article number 1970. doi: 10.3390/agronomy10121970.

[60] Schreiber, J.B. (2021). Issues and recommendations for exploratory factor analysis and principal component analysis. *Research in Social and Administrative Pharmacy*, 17(5), 1004-1011. doi: 10.1016/j.sapharm.2020.07.027.

[61] Sharma, S., Singh, P., Choudhary, O.P., & Neemisha. (2021). Nitrogen and rice straw incorporation impact nitrogen use efficiency, soil nitrogen pools and enzyme activity in rice-wheat system in north-western India. *Field Crops Research*, 266, article number 108131. doi: 10.1016/j.fcr.2021.108131.

[62] Shrestha, N. (2021). Factor analysis as a tool for survey analysis. *American Journal of Applied Mathematics and Statistics*, 9(1), 4-11. doi: 10.12691/ajams-9-1-2.

[63] Sulaiman, M.S., Abood, M.M., Sinnakaudan, S.K., Shukor, M.R., You, G.Q., & Chung, X.Z. (2021). Assessing and solving multicollinearity in sediment transport prediction models using principal component analysis. *ISH Journal of Hydraulic Engineering*, 27(1), 343-353. doi: 10.1080/09715010.2019.1653799.

[64] Sun, X., Ye, Y., Ma, Q., Guan, Q., & Jones, D.L. (2021). Variation in enzyme activities involved in carbon and nitrogen cycling in rhizosphere and bulk soil after organic mulching. *Rhizosphere*, 19, article number 100376. doi: 10.1016/j.rhisph.2021.100376.

[65] Tahir, M., Wei, X., Liu, H., Li, J., Zhou, J., Kang, B., Jiang, D., & Yan, Y. (2023). Mixed legume-grass seeding and nitrogen fertilizer input enhance forage yield and nutritional quality by improving the soil enzyme activities in Sichuan, China. *Frontiers in Plant Science*, 14, article number 1176150. doi: 10.3389/fpls.2023.1176150.

[66] Tan, X., Nie, Y., Ma, X., Guo, Z., Liu, Y., Tian, H., Megharaj, M., Shen, W., & He, W. (2021). Soil chemical properties rather than the abundance of active and potentially active microorganisms control soil enzyme kinetics. *Science of the Total Environment*, 770, article number 144500. doi: 10.1016/j.scitotenv.2020.144500.

[67] Tetteh, C.K. (2024). R&D, innovation and labour productivity: Evidence from the manufacturing sector in Ghana. *Scientific African*, 24, article number e02136. doi: 10.1016/j.sciaf.2024.e02136.

[68] Tian, P., Lian, H., Wang, Z., Jiang, Y., Li, C., Sui, P., & Qi, H. (2020). Effects of deep and shallow tillage with straw incorporation on soil organic carbon, total nitrogen and enzyme activities in Northeast China. *Sustainability*, 12(20), article number 8679. doi: 10.3390/su12208679.

[69] Wang, D., Lan, Y., Chen, W., Liu, Z., Gao, J., Cao, D., Wang, Q., Mazhang, C., & An, X. (2024). Response of bacterial communities, enzyme activities and dynamic changes of soil organic nitrogen fractions to six-year different application levels of biochar retention in Northeast China. *Soil and Tillage Research*, 240, article number 106097. doi: 10.1016/j.still.2024.106097.

[70] Watkins, M.W. (2021). *A step-by-step guide to exploratory factor analysis with SPSS*. New York: Routledge. doi: 10.4324/9781003149347.

[71] Watthier, M., Peralta Antonio, N., Gomes, J.A., Rocha, S.B.F., & Santos, R.H.S. (2020). Decomposition of green manure with different grass: Legume ratios. *Archives of Agronomy and Soil Science*, 66(7), 913-924. doi: 10.1080/03650340.2019.1644622.

[72] Yan, M., Pan, G., Lavallee, J.M., & Conant, R.T. (2020). Rethinking sources of nitrogen to cereal crops. *Global Change Biology*, 26(1), 191-199. doi: 10.1111/gcb.14908.

[73] Yang, J., Yang, W., Wang, F., Zhang, L.M., Zhou, B., Sarfraz, R., & Xing, S. (2020). Driving factor of soluble organic nitrogen dynamics in paddy soils: Structure equation modeling analysis. *Pedosphere,* 30(6), 801-809. doi: 10.1016/S1002-0160(18)60032-3.

[74] Yang, Y., Li, M., Wu, J., Pan, X., Gao, C., & Tang, D.W.S. (2022). Impact of combining long-term subsoiling and organic fertilizer on soil microbial biomass carbon and nitrogen, soil enzyme activity, and water use of winter wheat. *Frontiers in Plant Science*, 12, article number 788651. doi: 10.3389/fpls.2021.788651.

[75] Zhai, L., Wang, F., Yan, A., Liang, C., Wang, S., Wang, Y., & Xu, J. (2020). Pleiotropic effect of *GNP1* underlying grain number per panicle on sink, source and flow in rice. *Frontiers in Plant Science*, 11, article number 933. doi: 10.3389/fpls.2020.00933.

[76] Zhang, C., Zhao, Z., Li, F., & Zhang, J. (2022). Effects of organic and inorganic fertilization on soil organic carbon and enzymatic activities. *Agronomy*, 12(12), article number 3125. doi: 10.3390/agronomy12123125.

[77] Zhang, X., Zhu, B., Yu, F., & Cheng, W. (2021). Plant inputs mediate the linkage between soil carbon and net nitrogen mineralization. *Science of the Total Environment*, 790, article number 148208. doi: 10.1016/j.scitotenv.2021.148208.

[78] Zhao, Z., Zhang, C., Li, F., Gao, S., & Zhang, J. (2020). Effect of compost and inorganic fertilizer on organic carbon and activities of carbon cycle enzymes in aggregates of an intensively cultivated Vertisol. *Plos One*, 15(3), article number e0229644. doi: 10.1371/journal.pone.0229644.

[79] Zhou, S.M., Zhang, M., Zhang, K.K., Yang, X.W., He, D.X., Yin, J., & Wang, C.Y. (2020). Effects of reduced nitrogen and suitable soil moisture on wheat (*Triticum aestivum* L.) rhizosphere soil microbiological, biochemical properties and yield in the Huanghuai Plain, China. *Journal of Integrative Agriculture*, 19(1), 234-250. doi: 10.1016/S2095-3119(19)62697-3.

[80] Zhu, Y., Archontoulis, S.V., & Castellano, M.J. (2025). Quantifying the timing asynchrony between soil nitrogen availability and maize nitrogen uptake. *Plant and Soil*. doi: 10.1007/s11104-025-07352-3.

[81] Zusfahair, Z., Fatoni, A., Ningsih, D.R., & Riapanitra, A. (2021). Determination of Cu and Pb concentrations based on urease activity inhibition of *Durio zibethinus* L. seeds. *Molekul*, 16(2), 92-99. doi: 10.20884/1.jm.2021.16.2.736.

Каузальні залежності між продуктивністю пшениці й біологічною активністю, вмістом у ґрунті азоту

Олег Стасів
Доктор сільськогосподарських наук, директор
Інститут сільського господарства Карпатського регіону Національної академії аграрних наук
81115, вул. Грушевського, 5, с. Оброшине, Україна
https://orcid.org/0000-0003-3737-739X

Ангеліна Дубицька
Кандидат сільськогосподарських наук, провідний науковий співробітник
Інститут сільського господарства Карпатського регіону Національної академії аграрних наук
81115, вул. Грушевського, 5, с. Оброшине, Україна
https://orcid.org/0000-0002-5685-0237

Оксана Качмар
Кандидат сільськогосподарських наук, завідувач відділу
Інститут сільського господарства Карпатського регіону Національної академії аграрних наук
81115, вул. Грушевського, 5, с. Оброшине, Україна
https://orcid.org/0000-0002-0382-6030

Олександр Дубицький
Кандидат біологічних наук, старший науковий співробітник
Інститут сільського господарства Карпатського регіону Національної академії аграрних наук
81115, вул. Грушевського, 5, с. Оброшине, Україна
https://orcid.org/0000-0002-8293-4119

Оксана Вавринович
Кандидат сільськогосподарських наук, провідний науковий співробітник
Інститут сільського господарства Карпатського регіону Національної академії аграрних наук
81115, вул. Грушевського, 5, с. Оброшине, Україна
https://orcid.org/0000-0003-3466-1432

Анотація. Висвітлення компонентів каузальних залежностей продуктивності сільськогосподарських культур, опосередкованих N-, C-хімічними, біологічними індикаторами ґрунту (розміри пулів відповідних доступних форм, ферментативних активностей) у контексті теоретичних аспектів обігу ресурсів у ньому, є потенційно важливим для конструювання систем оціночних критеріїв здоров'я ґрунту з використавнням моделей машинного навчання (ML). Мета роботи полягала у відшуковуванні статистично валідних і змістовних координацій і каузальних зумовленостей маси зерна на площу посіву доступними формами N, C (неорганічний і легкогідролізований N – N_IN, N_EH; C лабільного гумусу – C_LH), *in situ* протеолітичною PA, *in vitro* уреазною UA активністю у асоціації з концепціями обігу зазначених органогенів у ґрунті під пшеницею озимою за прототипних екологічно безпечних систем добрив. Використано наступні методи: польові, аплікаційний (*in situ*), лабораторно-аналітичні хімічні й біохімічні, морфометричний, статистичні, ML-моделювання. З високою ймовірністю приріст GM був спричинений адитивним зростанням N_IN, N_EH, одночасним зменшенням *in situ* PA у спосіб, за якого вплив *in situ* PA на GM опосередковувався переліченими формами N. Статистично менш валідним було опосередкування дії співвідношення N_EH до N_IN на GM через in situ PA з, можливо, взаємно доповнюючими предетермінуваннями форм N, що втім, створювало основу для обгрунтування одночасних взаємно протилежних зумовеностей GM цими формами N й in situ PA. Правдоподібно, що C_LH й *in vitro* UA не брали участі у розвитку GM за даних експериментальних умов, проте не виключено, що друга серед цих змінних може більш значуще впливати на варіювання першої, ніж навпаки. Дослідження у такому руслі дозволять сформувати дискурс теоретичних передбачень стосовно удосконалення структури індексів якості ґрунту у напрямку розкриття аспектів закономірностей сприяння необхідному для розвитку стійкого сільського господарства балансу між характеристиками агроекосистемних послуг, з урахуванням хімічних і біохімічних властивостей обігу N і C у ґрунті, й біопродуктивності сільськогосподарських культур

Ключові слова: доступні форми азоту у ґрунті; вуглець лабільного гумусу; протеолітична й уреазна активність ґрунту; маса зерна пшениці озимої; експлораторний факторний аналіз; системи одночасних рівнянь; каузальні зумовленості
