On posets with nonnegative Tits form

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Let S be a finite poset (without the element 0) and let $\mathbb{Z}^{S\cup 0} = \{z = (z_i) \mid z_i \in \mathbb{Z}, i \in S \cup 0\}$ ($\mathbb{Z}$ denotes the integer numbers). The Tits quadratic form of S is the form $q_S : \mathbb{Z}^{S\cup 0} \to \mathbb{Z}$ defined by the equality

$$q_S(z) = z_0^2 + \sum_{i\in S} z_i^2 + \sum_{i<j,i,j\in S} z_i z_j - z_0 \sum_{i\in S} z_i.$$

The poset S with nonnegative (resp. positive) Tits form is said to be $\geq$-serial (resp. $>$-serial) if for any natural number m there is a poset $T \supset S$ with nonnegative (resp. positive) Tits form such that $|T \setminus S| = m$. We say that S is a sum of subposets A and B if $S = A \cup B$ and $A \cap B = \varnothing$.

Recall that a semichain is by definition a poset every element of which is comparable with all but at most one elements (the empty poset is a semichain).

Theorem 1. *Any $\geq$-serial poset is a sum of two semichains.*

A poset S is said to be $\geq$-critical (resp. $>$-critical) if the Tits form of any proper subposet of S is nonnegative (resp. positive), but that of S is not.

Theorem 2. *Any $\geq$-critical poset contains a $>$-critical one.*

By the method of (min, max)-equivalence of posets [1] we calculate the full list of $\geq$-critical posets (the $>$-critical posets were described in [2]).

References

1. V. M. Bondarenko. *On* (min, max)*-equivalence of posets and applications to the Tits forms*// Bulletin of the University of Kiev (Series: Physics & Mathematics), 2005, N1, P. 11–13.
2. V. M. Bondarenko, M. V. Styopochkina. (Min, max)*-equivalence of partially ordered sets and the Tits quadratic form*// Transactions of Institute of Mathematics of NAS of Ukraine, 2005, vol. 2, N3, P. 18–58 (In Russian).