On injective representations of posets and the quadratic Tits form

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Let S be a finite poset (without the element 0), *Inj S* the category of injtctive representations of S and $Rep_{inj}S$ the category of injtctive representations of *Inj S*. Denote by $\vec{S}$ the graph with the set of vertices S and the arrows (x, y), where x and y are adjacent and $x<y$ (i.e. there is no element z such that $x<z<y$).

Recall that the Tits quadratic form of *S* is by definition the form $q_S : \mathbb{Z}^{S \cup 0} \to \mathbb{Z}$ defined by the equality

$$q_S(z) = z_0^2 + \sum_{i \in S} z_i^2 + \sum_{i<j, i,j \in S} z_i z_j - z_0 \sum_{i \in S} z_i \, .$$

We prove the following theorems.

Theorem 1. *Let S be a nonselfdual poset such that the graph* $\vec{S}$ *is a disjoint union of chains. Then both the categories* $Rep_{inj} S$ *and* $Rep_{inj} S^{\mathrm{op}}$ *have (up to isomorphism) only finitely many indecomposable object if and only if the Tits form of S is positive definite.*

Theorem 2. *Let S be a nonselfdual poset of width w<3 such that the graph* $\vec{S}$ *has no cycles. Then both the categories* $Rep_{inj} S$ *and* $Rep_{inj} S^{\mathrm{op}}$ *have (up to isomorphism) only finitely many indecomposable object if and only if the Tits form of S is positive definite.*

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